淡江大學八十八學年度碩士班招生考試試題

系別：統計學系　　科目：基礎數學　　本試題共 2 頁

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1) (a) Determine whether the function f defined by

$$f(x)=\begin{cases}\dfrac{9x^2-3x^3}{x-3}, & for\ x\neq 3\\ 27, & for\ x=3\end{cases}$$

continuous? Why? (5%)

(b) Let $f(x)=\begin{cases}\dfrac{x^2-x-2}{x-2}, & if\ x>2\\ \dfrac{2x-1}{x-1}, & if\ x\le 2\end{cases}$

Is f a continuous function ? Why? (5%)

2) (a) State the mean-value theorem. (5%)

(b) Let f be a differentiable function with domain containing the interval $[a,b]$.

Suppose that $m\le f'(x)\le M,\ \forall x\in[a,b]$. Prove that

$$f(a)+m(x-a)\le f(x)\le f(a)+M(x-a),\ \forall x\in[a,b].$$ (7%)

3) Let $L:R^3\to R^3$, $L\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=\begin{bmatrix}u\\v\\w\end{bmatrix}$ be a function defined by

$$u=3x+2y-z$$
$$v=x-y+z$$
$$w=x+2y-z$$

(a) Find a matrix A such that $L\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=A\begin{bmatrix}x\\y\\z\end{bmatrix}$. (4%)

(b) Prove that L invertible. Find the explicit equations for the corresponding inverse function L^{-1}. (8%)

(c) Compute the Jacobian J for L and the Jacobian j for L^{-1}, and show that $J\cdot j=1$. (6%)

4) Let $T=T(x,y), U=U(x,y)$ be such that $e^T+\ln U=2x+y$ *and* $\ln T+e^U=x+2y$.

Find $\dfrac{\partial T}{\partial x}$ *and* $\dfrac{\partial U}{\partial x}$. (8%)

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5) Find the following integrals: (18%)

(a) $\int_0^\infty x^3 e^{-\frac{x}{3}} dx$　　(b) $\int_0^1 x^3 (1-x)^3 dx$.

(c) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{1}{2}\left\{\left(\frac{x-1}{2}\right)^2+\left(\frac{y-3}{4}\right)^2\right\}} dx\,dy$

6) Let $L: R^4 \to R^4$ be a linear transformation defined by $L(\vec{x}) = A\vec{x}$, where

$$A=\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & 0 \\ \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & -\frac{3}{\sqrt{12}} \end{bmatrix}.$$

(a) Prove that L is orthogonal. (8%)

(b) Find the determinant of A. (4%)

(c) Find the inverse of A. (4%)

7) Let $A=\begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$ and I be the 3×3 identity matrix.

(a) Find the null space of A. What are the rank and nullity of A? (8%)

(b) Find the eigenvalues of A^3, A^{-1}, *and* $A+3I$. (10%)