淡江大學八十七學年度碩士班入學考試試題

系別： 統計學系　　　　科目： 基礎數學(含微積分、線性代數)

(1) Test for convergence $1 + 2\gamma + \gamma^2 + 2\gamma^3 + \gamma^4 + 2\gamma^5 + \cdots$ where
(a) $\gamma = 2/3$, (b) $\gamma = -2/3$, (c) $\gamma = 4/3$. (10%)

(2) Find constants a and b for which
$$F(a,b) = \int_0^\pi \{\sin x - (ax^2 + bx)\}^2 \, dx$$
is a minimum. (10%)

(3) Find a unit vector tangent to the space curve $x = t$, $y = t^2$, $z = t^3$ at the point where $t = 1$. (10%)

(4) Let $f(x,y) = \begin{cases} xy\left(\dfrac{x^2 - y^2}{x^2 + y^2}\right), & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Compute (a) $f_x(0,0)$, (b) $f_y(0,0)$, (c) $f_{xx}(0,0)$, (d) $f_{yy}(0,0)$, (e) $f_{xy}(0,0)$, (f) $f_{yx}(0,0)$. (10%)

(5) Find $\displaystyle \lim_{n\to\infty} \left\{ \frac{n}{n^2 + 1^2} + \frac{n}{n^2 + 2^2} + \cdots + \frac{n}{n^2 + n^2} \right\}$. (10%)

(6) Prove that $\displaystyle \int_0^{1/2} \frac{\cos \pi x}{\sqrt{1 + x^2}} \, dx \leq \frac{1}{4} \tan^{-1} \frac{1}{2}$. (10%)

(7) Evaluate $\displaystyle \iint_R \sqrt{x^2 + y^2} \, dx \, dy$ where R is the region in the xy plane bounded by $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. (10%)

(8) Find an orthogonal matrix P such that $P^{-1}AP$ is diagonal, where

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ -1 & 2 & 5 \end{bmatrix} \quad (10\%)$$

(9) Find the projection of $U = (2, -3, 1)$ on $d = (1, -1, 3)$ and express $U = U_1 + U_2$ where U_1 is parallel to d and U_2 is orthogonal to d. (10%)

(10) Find a basis for the null space of $A = \begin{bmatrix} 1 & -2 & 0 & 1 \\ -1 & 2 & 0 & 1 \\ 2 & -4 & 1 & 0 \end{bmatrix}$. (5%)

(11) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ be a fixed matrix in M_{22}, and let

$$U = \{ X \text{ in } M_{22} \mid AX = XA \}$$

Show that U is a subspace of M_{22}, where M_{22} is the set of all 2×2 matrices. (5%)