143-1

淡江大學九十四學年度碩士班招生考試試題

系別：統計學系　　　　科目：機　率　論

准帶項目請打「V」	
	簡單型計算機

本試題共 1 頁

1.

(i) For three events A, B, and C, suppose that $P(A \cap B) = P(A \cap C)$ and $P(B \cap C) = 0$. Then show that $P(A \cup B \cup C) = P(A) + P(B) + P(C) - 2P(A \cap B)$. (10 points)

(ii) For any two events A and B, show that $P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A \cap B)$. (10 points)

2. If X is a Poisson random variable with parameter λ. Show that $\frac{P(X=i+1)}{P(X=i)} = \frac{\lambda}{i+1}$, and then it can be represented as $P(X = i+1) = \frac{\lambda}{i+1} P(X = i)$. (10 points)

3. For a nonnegative random variable Y, show that $E(Y) = \int_0^\infty P(Y > y)dy$. (10 points)

4. Use polar coordinates transformation to show that $I = \int_{-\infty}^{\infty} e^{-y^2/2} dy = \sqrt{2\pi}$. (10 points)

5. Prove the equation $Var(X) = E[Var(X|Y)] + Var[E(X|Y)]$. (10 points)

6. If X is a binomial random variable with parameter (n, p), where $0 < p < 1$.

(i) Show that the kth moment $E(X^k) = npE[(Y+1)^{k-1}]$, where Y is a binomial random variable with parameters $(n-1, p)$. (10 points)

(ii) Use (i) to show that $E(X) = np$ and $Var(X) = np(1-p)$. (10 points)

7. If X and Y are independent gamma random variables with respective parameters (α_1, λ) and (α_2, λ), then show that $X + Y$ is a gamma random variable with parameters $(\alpha_1 + \alpha_2, \lambda)$. The probability density function of gamma random variable with parameters (α, λ) is $f_X(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$, $x > 0$. (10 points)

8. If the r.v. X has the p.d.f. $f_X(x) = \frac{1}{\sqrt{2\pi}} x^{-2} e^{-\frac{1}{2x^2}}$, $x \in R$, show that the random variable $Y = \frac{1}{X}$ has a standard normal distribution. (10 points)