淡江大學九十三學年度碩士班招生考試試題

系別：統計學系　　科目：機　率　論

准帶項目請打「○」否則打「×」
簡單型計算機
○

[1] Let $\{A_j, j = 1,\ldots,5\}$ be a partition of S and suppose that $P(A_j) = j/15$ and $P(A|A_j) = (5\text{-}j)/15, j = 1,\ldots,5$. Compute the probabilities $P(A_j|A), j = 1,\ldots,5$. (20 %)

[2] If X is an r.v. distributed as $N(\mu,\sigma^2)$, find the value of c (in terms of μ and σ) for which $P(X < c) = 2 - 9P(X > c)$. (15 %)

[3] Let the r.v. X be distributed as Poisson with parameter λ and define the r.v. Y as follows:

$Y = X$ if $X \geq k$ (a given positive interger) and $Y = 0$ otherwise.

Find (i) $P(Y = y), y = k, k+1,\ldots$; (8 %) (ii) $P(Y = 0)$. (7 %)

[4] Let X and Y be r.v.'s whose joint p.d.f. f is given by $f(x,y) = cxyI_{(0,2)\times(0,5)}(x,y)$.

Determine the constant c and compute the following probabilities:

(a) $P(\frac{1}{2} < X < 1, 0 < Y < 3)$; (b) $P(1 < X < 2, Y > 5)$; (c) $P(X > Y)$. (15 %)

[5] By using the Cauchy-Schwarz inequality of **Probability Thory** to show that

$(a_1^2 + a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2) \geq (a_1b_1 + a_2b_2 + \cdots + a_nb_n)^2$. (5 %)

[6] Let X be an r.v. such that $P(X=j) = (\frac{1}{2})^j, j = 1,2,\ldots\ldots$

(1) Compute $EX, E[X(X-1)]$; (10 %)

(2) Use (1) in order to compute $\sigma^2(X)$. (10 %)

[7] Let $X_1, X_2, \cdots, X_n$ be a random sample from the exponential distribution with the probability density function as given by

$$f(x) = \frac{1}{\theta}\exp(-\frac{x}{\theta}),\ x > 0,\ \theta > 0.$$

Then the corresponding order statistics are $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$. Find

$E(\tilde{X}_{(j)} - X_{(j)})$ and $Var(\tilde{X}_{(j)} - X_{(j)})$, $1 < r < j < n$, where

$\tilde{X}_{(j)} = X_{(r)} + \theta_*[(n-r)^{-1} + (n-r-1)^{-1} + \cdots + (n-j+1)^{-1}]$ and

$\theta_* = [X_{(1)} + X_{(2)} + \cdots + X_{(r)} + (n-r)X_{(r)}]/r$. (10 %)