淡江大學九十二學年度碩士班招生考試試題

系別：統計學系　　科目：機　率　論

准帶項目請打「○」否則打「×」
簡單型計算機

1) State the following theorems in probability theory: (24%)
   (a) Central Limit Theorem
   (b) Weak Law of Large Numbers
   (c) Chebyshev's Inequality
   (d) Cauchy-Schwarz Inequality

2) Let X,Y be two random variables with joint probability density function (p.d.f.) given by

$$f_{X,Y}(x,y)=\begin{cases}2, 0<x<y<1\\ 0, otherwise\end{cases}$$

   (a) Find the marginal p.d.f's f_X, f_Y of X and Y, respectively. (8%)
   (b) Find the conditional probability $P\left(-1<Y<\frac{1}{2}\,|\,X=\frac{1}{4}\right)$. (6%)
   (c) Find the covariance, $Cov(X,Y)$, of X and Y. (8%)

3) Let X,Y be two random variables with joint p.d.f. given by

$$f_{X,Y}(x,y)=\begin{cases}\frac{x+y}{21}, x=1,2,\ y=1,2,3\\ 0, \quad otherwise\end{cases}$$

   (a) Find the marginal p.d.f. f_X of X. (4%)
   (b) Find $E(Y\,|\,X=1)$ and $Var(Y\,|\,X=1)$. (12%)

4) Let X_1,X_2,X_3,X_4 be a random sample from a distribution having p.d.f. given by $f(x)=\begin{cases}\frac{1}{4}, x=1,2,3,4\\ 0, otherwise\end{cases}$. Let $Y_1=\min\{X_1,X_2,X_3,X_4\}$ and $Y_4=\max\{X_1,X_2,X_3,X_4\}$.
   (a) Find the distribution function of X_1. (4%)
   (b) Find the p.d.f. of Y_1. (8%)
   (c) Find the p.d.f. of Y_4. (8%)

5) Let X_1,X_2 be random variables distributed as normal $N(0,\sigma^2)$ and let $T=X_1^2+X_2^2$.
   (a) Find the distribution of T. (10%)
   (b) Find the mean and variance of T. (8%)