淡江大學九十學年度碩士班招生考試試題

系別：統計學系　　科目：機　率　論

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本試題雙面印製

1. Let C_1 and C_2 be independent events with $P(C_1) = 0.6$ and $P(C_2) = 0.3$. Compute (a) $P(C_1 \cup C_2)$ (b) $P(C_1 \cap C_2)$ (c) $P(C_1^* \cap C_2^*)$ (d) $P(C_1 \cup C_2^*)$, where A^* is called the complement of A. (20%)

2. Let X be a random variable such that $\Pr(X \leq 0)=0$ and let $\mu = \mathrm{E}(X)$ exist. Show that $\Pr(X \geq 2\mu) \leq \frac{1}{2}$. (10%)

3. Let X and Y have the joint probability density function (p.d.f.) $f(x,y) = 6(1-x-y), 0 < x, 0 < y, x+y < 1$, and zero elsewhere. Compute $\Pr(2X + 3Y < 1)$ and $\mathrm{E}(XY + 2X^2)$. (10%)

4. Let X be a random variable such that $\mathrm{E}(X^{2m})=\frac{(2m)!}{(2^m m!)}$, $m = 1, 2, 3, \ldots$ and $\mathrm{E}(X^{2m-1})=0$, $m = 1, 2, 3 \ldots$. Find the moment generating function (m.g.f.) and the p.d.f. of X. (20%)

5. If the independent variables X_1 and X_2 have means μ_1, μ_2 and variances σ_1^2, σ_2^2, respectively. Find the mean and variance of the product $Y = X_1 X_2$. (10%)

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from a distribution with mean mu and variance σ^2. Consider the second differences

$$Z_j = X_{j+2} - 2X_{j+1} + X_j, \qquad j = 1, 2, \cdots, n-2.$$

Compute the variance of the average, $\sum_{j=1}^{n-2} \frac{Z_j}{n-2}$, of the second differences. (10%)

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7. Let Y_n denote the nth order statistic of a random sample of size n from a uniform distribution on the interval $(0,\theta)$. Prove that $Z_n = \sqrt{Y_n}$ converges in probability to $\sqrt{\theta}$. (10%)

8. Let $Y_1, Y_2, \cdots, Y_n$ be the order statistics of a sample of size n from an exponential distribution with parameter $\mu = 1$. Find that $\mathrm{E}[\prod_{i=1}^{n}(n-i+1)(Y_j - Y_{j-1})]$, where $Y_0 = 0$. (10%)