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淡江大學八十九學年度碩士班招生考試試題

系別：統計學系　　科目：機　率　論

本試題共 / 頁

(10%) 1. Let $Y_1 < Y_2 < \cdots < Y_5$ be the order statistics of a random sample of size 5 from a distribution that has the p.d.f. (probability density function) $f(x) = 1, 0 < x < 1$, zero elsewhere. Compute $\Pr(Y_1 < \frac{1}{5}, Y_5 > \frac{3}{5})$.

(10%) 2. Let X and Y be random variables with means μ_1, μ_2; variances σ_1^2, σ_2^2; and correlation coefficient ρ. Find the correlation coefficient of $W = aX + b$, $a > 0$, and $Z = cY + d$, $c > 0$.

(10%) 3. Let X be a random variable such that $E(X^m) = (m+1)!2^m$, $m = 1, 2, 3, \cdots$. Determine the m.g.f. (moment generating function) and the distribution of X.

(10%) 4. (a) If $E[X] = \mu$, $Var(X) = \sigma^2$, then for $a > 0$, show that $\Pr(X \leq \mu - a) \leq \frac{\sigma^2}{\sigma^2 + a^2}$.

(b) A set of 200 people, consisting of 100 men and 100 women, is randomly divided into 100 pairs of 2 each. Give an upper bound to the probability that at most 30 of these pairs will consist of a man and a woman. (Hint: use (a))

(10%) 5. Let the r.v.s X_1, X_2, X_3 be jointly distributed with p.d.f. f given by

$$f(x_1, x_2, x_3) = \frac{1}{4} I_A(x_1, x_2, x_3),$$

where

$$A = \{(1,0,0), (0,1,0), (0,0,1), (1,1,1)\}.$$

Then, show that (i) $X_i, X_j, i \neq j$, are independent; (ii) X_1, X_2, X_3 are dependent.

(20%) 6. Each of four persons fires one shot at a target. Let C_k denote the event that the target is hit by person k, $k = 1, 2, 3, 4$. If C_1, C_2, C_3 and C_4 are independent and if $Pr(C_1) = Pr(C_2) = 0.7$, $Pr(C_3) = 0.9$, and $Pr(C_4) = 0.4$, compute the probability that (a) all of them hit the target; (b) exactly one hits the target; (c) no one hits the target; (d) at least one hits the target.

(20%) 7. Let X have a gamma distribution with p.d.f.

$$f(x) = \frac{1}{\beta^2} x \exp(-\frac{x}{\beta}),\ 0 < x < \infty,$$

zero elsewhere. If $x = 2$ is the unique mode of the distribution, find the parameter β and $Pr(X < 9.49)$.

(10%) 8. Let Y be $\chi^2(n)$. What is the limiting of $Z = \sqrt{Y} - \sqrt{n}$?