淡江大學八十八學年度碩士班招生考試試題

系別：統計學系　　科目：機率論　　本試題共　乙　頁

(1) A point is chosen at random from the triangle whose vertices are at (0,0), (0,2), and (2,0); X is the sum of the two coordinates of the point.

a. Find the cumulative distribution function of X. (5%)

b. Find the probability density function of X. (5%)

c. Find $P(X \leq 1)$. (5%)

(2) Let $X_1, X_2, \cdots, X_9$ be independent random variables, all having the Poisson distribution with parameter λ.

a. What is the probability that $X_1 + X_2 + \cdots + X_9 = 0$? (5%)

b. What is the probability that $X_1 + X_2 + \cdots + X_9 = 1$? (5%)

c. What is the probability that $X_1 + X_2 + \cdots + X_9 = 2$? (5%)

d. From the answers to parts a, b, c, guess the distribution of $X_1 + X_2 + \cdots + X_9$. (5%)

(3) a. Let μ_k be the kth moment $E(X^k)$ of X. Show that $\mu_{2n} \geq (\mu_n)^2$ for any positive integer n. (10%)

b. The possible values of X are the positive integers, and $P(X = k) = kp^2q^{k-1}$ for $k = 1, 2, 3, \cdots$. (As always, p and q are positive constants and $p + q = 1$.) Find $E(X)$. (5%)

(4) a. Let X and Y be independent, each with the same distribution, whose moment generating function is $m(s) = \sqrt{1/(1-4s)}$ (for $s < \frac{1}{4}$). What is the name of the distribution of $X + Y$? (5%)

b. If Z is a unit normal random variable and if Y is defined by $Y = A + BZ + CZ^2$, show that $\rho(Y, Z) = \frac{B}{\sqrt{B^2+2C^2}}$. (5%)

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(5) If $X_1, X_2, \cdots, X_n$ are independent and identically distributed random variables having uniform distributions over (0,1), find

a. $E[max(X_1, \cdots, X_n)]$; (5 %)

b. $E[min(X_1, \cdots, X_n)]$. (5 %)

(6) a. If Y is uniformly distributed over (0,5) what is the probability that the roots of the equation $4x^2 + 4xY + Y + 2 = 0$ are both real ? (5 %)

b. If X is a Poisson random variable with mean λ, show that for $i < \lambda$, $P(X \leq \lambda) \leq \frac{e^{-\lambda}(e\lambda)^i}{i^i}$. (5 %),

c. If $E[X] < 0$ and $\theta \neq 0$ is such that $E[e^{\theta X}] = 1$, show that $\theta > 0$. (5 %)

(7) An engineering system consisting of n components is said to be a k-out-of-n system ($k \leq n$) if the system functions if and only if at least k of the n components function. Suppose that all components function independently of each other.

a. If the ith component functions with probability p_i, $i = 1, 2, 3, 4, 5$, compute the probability that a 3-out-of-5 system functions. (5 %)

b. Repeat part a. for a 2-out-of-4 system. (5 %)

c. Repeat for a k-out-of-n system when all the p_i equal p (that is, $p_i = p, i = 1, 2, 3, \cdots, n$). (5 %)

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