

系列：管理科學研究所

科目：微 積 分

本試題共 / 頁， 9 大題

一、Find the derivative $\frac{df(x)}{dx}$ of each of the following functions (18%)

(1) $f(x) = x^{\sqrt{x+1}}$ (2) $f(x) = \ln \ln \left(\frac{x+1}{x} \right)$

(3) $x + f(x) = 4(x - f(x))$

二、Evaluate each of the following integrals (24%)

(1) $\int \frac{dx}{\sqrt{x}(1-\sqrt{x})}$ (2) $\int_0^1 \frac{e^x}{\sqrt{e^x+1}} dx$

(3) $\int_0^{\frac{\pi}{4}} \sec \theta d\theta$ (4) $\int_0^1 \left(\int_{y^2}^1 ye^{x^2} dx \right) dy$

三、 $\lim_{n \rightarrow \infty} \frac{1}{n^2} \left\{ \sqrt{\frac{n+1}{n}} + 2\sqrt{\frac{n+2}{n}} + 3\sqrt{\frac{n+3}{n}} + \dots + n\sqrt{2} \right\} = ?$ (8%)

四、Find the area of the region bounded by $y = x^2 + 1$, $y = x$, $x = 0$ and $x = 1$. (8%)

五、Find the minimum value of the function $f(x) = 2x^3 - 15x^2 + 24x + 19$ for $x \geq 0$. (8%)

六、What rational number has the decimal expansion $0.1212\overline{12}$? (8%)

七、Find the antiderivative $F(x)$ of the function $f(x) : 4x^3 + \frac{2}{\sqrt{x}} + 4e^{-2x} + \frac{y}{x}$ for

which $F(1) = \frac{3}{e^2} + 8$. (8%)

八、Let $D(x) = \frac{2500}{x+50}$ be the demand curve and $S(x) = 0.01x^2$ be the supply curve.

Find the consumer's surplus and supplier's surplus with respect to the equilibrium point (50, 25). You may use $\ln 2 = 0.693$. (8%)

九、A college bookstore expects to sell 1000 pen and pencil sets during the next year. Its supplier is able to fill an order immediately, the cost of placing each order is \$25. The bookstore's average storage cost is \$0.8 per set per year. Assuming that the pen and pencil sets sell at a uniform rate, how many sets should the bookstore order in each shipment that minimize the total cost. (10%)