

系別：管理科學研究所

科目：微 積 分

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

一. Solve the following problems :
 (20%)

$$(i) \lim_{x \rightarrow 1^-} \left(\frac{1}{1-x} + \frac{1}{\ln x} \right) = ? , (ii) \lim_{x \rightarrow \infty} (1+x)^x = ? , (iii) \int \frac{1}{4-x^2} dx = ? , (iv) \int \frac{dx}{\sqrt{4-x^2}} = ?$$

二. Prove : $\frac{d}{dx}(x^n) = nx^{n-1}$ where n is a real number
 (15%)

三. Use integration to find the volume under surface $f(x,y) = 12xy$

(15%) above the region $R = \{(x,y) \mid y = \sqrt{x} \text{ and } y = x^2\}$

四. Find the Taylor series at $x=0$ for function $f(x) = e^x$ (10%) and estimate $e^{\frac{1}{2}}$ by using the first four terms of the Taylor series (5%).
 (15%)

五. Find the critical points, extreme values, points of inflection, and then

(20%) sketch the graph of function $f(x) = \frac{x}{1+x^2}$.

六. A company's output is given by the Cobb-Douglas production
 (15%) function $P = 600 L^{\frac{2}{3}} K^{\frac{1}{3}}$ where L and K are the numbers of units
 of labor and capital. Each unit of labor costs the company \$40,
 and each unit of capital costs the company \$100. If the company
 has a total of \$3000 for labor and capital, how much of each
 should it use to maximize production?