152-1

淡江大學96學年度碩士班招生考試試題

系別：管理科學研究所　　　　科目：統　計　學

准帶項目請打「V」	
V	簡單型計算機

本試題共 2 頁 P1.

本試題雙面印製

※ **All questions are individually.**

1. The following problems are descriptive statistics. (5 points each)

(1) Discuss and distinguish between discrete and continuous values.

(2) A motor car traveled 3 consecutive miles, the first mile at $x_1 = 35$ miles per hour (mph), the second at $x_2 = 48$ mph, and the third at $x_3 = 40$ mph. Find the average speed of the car in miles per hour.

(3) What can be said about a sample of observations whose standard deviation is zero ?

2. The following problems belong to probability distributions (5 points each)

(1) Convert the function $g(x) = 1/x^2$ to a probability density function $f(x)$ of a discrete random variable X. Let $f(x)$ have a non-zero value when x is a positive integer, and $f(x) = 0$ when x is not a positive integer.

(2) Defects occur along the length of a cable at an average of 6 defects per 4000 feet. Assume that the probability of k defects in t feet of cable is given by the

probability mass function : $\Pr(k \text{ defects}) = \dfrac{e^{-\frac{6t}{4000}}\left(\frac{6t}{4000}\right)^k}{k!}$ for $k = 0, 1, 2, \ldots$

Find the probability that a 3000-foot cable will have at most two defects.

(3) Consider the function $g(x) = (1 + x^2)^{-1}$. Determine a constant k such that $f(x) = k(1 + x^2)^{-1}$ is a proper probability density for $-\infty < x < \infty$. Find $F(x) = \Pr(X \le x)$ if X is distributed with density function $f(x)$.

3. The following problems are concerning the Binomial distribution (5 points each)

(1) Cars coming to a dead-end intersection can either turn left or turn right. The probability of any car turning left is 0.7. Suppose 4 cars choose a turning direction independent of one another. What is the probability of observing exactly three cars that make a right turn ?

(2) Find the coefficient of $x_1^5 x_2^3 x_3^2$ in the expansion of $(x_1 + x_2 + x_3)^{10}$.

152-2

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本試題共 2 頁 P2.

4. (1) (7 points) Suppose X_1, $X_2, \ldots$, X_n are independent Bernoulli random variables, that is, $\Pr(X_i = 0) = 1 - p$ and $\Pr(X_i = 1) = p$ for $i = 1, 2, \ldots, n$.

   What is the distribution of $Y = \sum_{i=1}^{n} X_i$ with proof ?

   (2) (8 points) Suppose X has a normal distribution with mean 0 and variance 1. What is the distribution of $Y = X^2$ with proof ?

5. (1) (5 points) Suppose X takes on the values 0, 1, 2, 3, 4, 5 with probabilities

   p_0, p_1, p_2, p_3, p_4 and p_5. If $Y = g(X) = (X-2)^2$,

   what is the distribution of Y ?

   (2) (10 points) Suppose X has the Poisson distribution with parameter λ.

   Let $Y = \begin{cases} 1 & \text{if } X \text{ is even} \\ -1 & \text{if } X \text{ is odd} \end{cases}$, Find the distribution of Y.

6. (1) (7 points) Let X_1, $X_2, \ldots$, X_n be a random sample from a normal distribution with mean μ and variance σ^2. Let $(\theta_1, \theta_2) = (\mu, \sigma)$. Estimate the parameters μ and σ by the method of moments.

   (2) (8 points) Consider the probability density function

   $$f(x;\theta) = \begin{cases} 1, & \theta - \frac{1}{2} \le x \le \theta + \frac{1}{2}, \quad -\infty < \theta < \infty \\ 0, & elsewhere \end{cases}$$

   Find a maximum likelihood statistic for the parameter θ.

7. (1) (7 points) We desire to perform an experiment to determine whether surface finish has an effect on the endurance limit of steel. There exists a theory that polishing increases the average endurance limit. If in our experiment we wish to detect that polishing fails to have an effect with a probability of 0.99 ($\alpha = 0.01$) and we also wish to detect a change of 7500 units by a probability of at least 0.9, then if it is known that the standard deviation of the endurance limit of the steel is 4000 units, what sample size would be needed to carry out this experiment ? (provide critical values $z_{0.01} = 2.33$ and $z_{0.10} = 1.28$)

   (2) (8 points) Let X possess a Poisson distribution with mean μ,

   i.e. $f(x,\mu) = e^{-\mu}\frac{\mu^x}{x!}$, Suppose we want to test the null hypothesis $H_0: \mu = \mu_0$ against the alternative hypothesis, $H_1: \mu = \mu_1$, where $\mu_1 < \mu_0$. Find the best critical region for this test.