淡江大學九十二學年度碩士班招生考試試題

系別：管理科學研究所　　　　科目：統　計　學

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本試題雙面印製

1. Given the data set from the following stem-and-leaf display

2	3 7
3	2 2 3
4	1 2
5	0

Find (1) the sample variance, S^2. (5 points)

(2) the 7th decile, D_7. (5 points)

2. Consider the distribution defined by the following distribution function:

$$F(x) = \begin{cases} 0 \quad , & if \ x < 0 \\ 1 - pe^{-x} \quad , & if \ x \geq 0 \quad for \ 0 < p < 1 \end{cases}$$

This distribution is partly discrete and partly continuous.

(1) Find the moment generating function of X. (10 points)

(2) Use the above (1) to find variance of X. (10 points)

3. Suppose random samples $X_1, \ldots, X_n$ are independent from the same distribution with finite mean μ and variance σ^2.

Show that the sample mean $\overline{X}$ is the B.L.U.E. (Best Linear Unbiased Estimator) for μ. (10 points)

4. Let the joint density function of X and Y be given by

$$f(x, y) = \begin{cases} e^{-x/y} e^{-y}, & 0 < x < \infty, \ 0 < y < \infty \\ 0, & otherwise \end{cases}$$

Find $P(X > 1 \mid Y = 1)$. (10 points)

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5. Let X_1 and X_2 be independent random variables with the following Poisson p.d.f.'s (probability density functions):

$$f_{X_i}(x_i) = e^{-\lambda_i}\frac{\lambda_i^{x_i}}{x_i},\quad x_i = 0,1,2,\ldots\ ;\ \lambda_i > 0\ ,\ i = 1,2$$

Find the joint p.d.f. of $Y_1 = X_1$ and $Y_2 = X_1 + X_2$. (10 points)

6. Let $X_1, X_2, \ldots$ be i.i.d.(identically independent distributions) $U(0,\ \theta)$ r.v.'s(random variables). (i.e. p.d.f. of X_i is $f(x|\theta) = \frac{1}{\theta} I_{(0<x<\theta)}$)

Let $Y_1 = \min(X_1, \ldots, X_n)$

Find the limiting distribution of $Z_n = nY_1$ as $n \to \infty$. (10 points)

7. A radioactive counting rate experiment was performed on five specimens of radium. The five different specimens were subjected to a counter with five shielding methods in various orders.
   (1) Discuss a suitable design with layout and write an appropriate model stating all assumptions. (10 points)
   (2) Give an ANOVA(Analysis of Variance) stating the sources and degrees of freedom only. (5 points)

8. Let r_s be the statistic of the Spearman Rank Correlation Test.

Show that $r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$, where d_i = the difference between two ranks of x_i and y_i, $i = 1,2,\ldots,n$. (i.e. $d_i = R(x_i) - R(y_i)$) (15 points)