淡江大學八十七學年度碩士班入學考試試題

系別： 產業經濟學系　　　科目： 計量經濟學

本試題共 2 頁

不得在本試題紙上作答，否則不予計分

1. Suppose you were given a random sample of 5 observations, X_1, X_2, X_3, X_4, and X_5, which were drawn independently from a normal distribution $N(\mu, \sigma^2)$. Consider the following two estimators of μ:

$$\hat{\mu}_1 = (\sum_{i=1}^{4} X_i)/8 + X_5/2,$$

$$\hat{\mu}_2 = (\sum_{i=1}^{4} X_i)/5 + X_5/5.$$

Are the two estimators unbiased? (10%)

2. Consider the following standard bivariate regression model:

$$Y_t = \beta_1 + \beta_2 X_t + u_t, t = 1, 2, \ldots, T.$$

The standard assumptions of the ordinary least squares regression model are assumed to hold. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be the OLSE of β_1 and β_2, respectively. Also, let $\overline{X}$ denote the sample mean: $\overline{X} \equiv (\sum_{t=1}^{T} X_t)/T$.

(i) Prove that $V(\hat{\beta}_2) = \sigma^2 / [\sum_{t=1}^{T} (X_t - \overline{X})^2]$. (10%)

(ii) Consider an alternative estimator $\beta_2^* = (Y_T - Y_1)/(X_T - \overline{X})$. Find $V(\beta_2^*)$ and compare it with $V(\hat{\beta}_2)$. Is β_2^* more efficient than $\hat{\beta}_2$? (10%)

3. For each of the following five models, examine whether β_1 and β_2 can be estimated by ordinary least squares. If yes, indicate how and describe the regression equation, the dependent variable, and the independent variable. If not, why not? (15%)

(i) $Y = \beta_1 + \beta_2 X + \beta_3 X^2 + u$

(ii) $Y = \beta_1 + \beta_2 X + uX$

(iii) $Y = \beta_1 e^{\beta_2 X + u}$

(iv) $Y = \beta_1 + e^{\beta_2 X + u}$

(v) $Y = \beta_1 X^{\beta_2} e^{u}$

4. Consider the following standard bivariate regression model:

$$Y_i = \beta_1 + \beta_2 X_i + u_i, i = 1,2,\ldots,N.$$

The standard assumptions of the ordinary least squares regression model are assumed to hold. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be the ordinary least squares estimators of β_1 and β_2,respectively.

(i) Show that $E(u_i|X_i)=0$ implies $E(X_i u_i)=0$. (5%)

(ii) Let $\hat{u}_i$ denote the residual. The sample statistics corresponding to $E(u_i)=0$ and $E(X_i u_i)=0$ are $(1/N)(\sum_{i=1}^{N}\hat{u}_i)$ and $(1/N)\sum_{i=1}^{N}(X_i\hat{u}_i)$, respectively. Find the normal equations by using these two sample statistics. (5%)

(iii) Solve the normal equations in (ii) for $\hat{\beta}_1$ and $\hat{\beta}_2$. (10%)

(iv) Suppose $N=3, X_1=5, X_2=3, X_3=0, Y_1=-3, Y_2=-1,$ and $Y_3=2$. Find $\hat{\beta}_1$ and $\hat{\beta}_2$ by using the solutions in (iii). (10%)

5. Is the following statement true, false or uncertain? Explain.
"According to the Gauss-Markov Theorem, the ordinary least squares estimators have the smallest variances than any other estimators." (5%)

6. Consider two models:

Model A: $Y = \beta_1 + \beta_2 X_2 + u$

Model B: $Y = \beta_1 + \beta_2 X_2 + \beta_3 X_3 + u$

Let $\hat{\beta}_2^A$ and $\hat{\beta}_2^B$ denote the OLSE of β_2 obtained from model A and model B, respectively.

(i) Is the following statement true, false or uncertain? Explain.
"The R^2 of model A is smaller than the R^2 of model B." (5%)

(ii) Is the following statement true, false or uncertain? Explain.
"If X_2 and X_3 are not correlated, then $\hat{\beta}_2^A = \hat{\beta}_2^B$." (5%)

(iii) In model B, describe how you would test the hypothesis $\beta_2 = \beta_3$ versus the hypothesis $\beta_2 \neq \beta_3$. (10%)