

淡江大學 96 學年度碩士班招生考試試題

系別：產業經濟學系

科目：微 積 分

准帶項目請打「V」	
✓	簡單型計算機

本試題共 / 頁

1. A function  $f(x)$  defined on the whole real line satisfies the following conditions:

$$f(0) = 0; \quad f(2) = 2; \quad \lim_{x \rightarrow +\infty} f(x) = 0$$

$$f'(x) = k(2x - x^2)e^{-x} \quad \text{for some positive constant } k.$$

a) Determine the intervals on which  $f$  is increasing and decreasing and the location of any local maximum and minimum values of  $f$ . [10 points]

b) Determine the intervals on which  $f$  is concave upward or downward, and the  $x$ -coordinates of any inflection points of  $f$ . [10 points]

c) Determine  $\lim_{x \rightarrow -\infty} f(x)$ . [10 points]

2. Let  $f(x, y) = 3xy^2 + 2x$  where  $x(t) = -3t^2$  and  $y(t) = 4t^3 + t$

a) Use the Chain Rule to find a general expression for the rate of change of the composite  $f(x(t), y(t))$  with respect to  $t$ . [10 points]

b) Use substitution and direct differentiation to compute the rate of change of the composite  $f(x(t), y(t))$  with respect to  $t$ . Compare this answer with your answer to part a). [10 points]

3. Find the maximum and minimum of  $f(x, y, z) = x + y + z^2$  subject to  $x^2 + y^2 + z^2 = 1$  and  $y = 0$ . [20 points]

4. a) Find the indefinite integral of  $(x^2 + 2x + 4)^{1/2}(x + 1)$ . [5 points]

b) Suppose the commodity  $Q$  has inverse demand function  $p = 3q^{-1/2}$  and that presently 100 units are being sold. What is the commodity's consumer surplus? [10 points]

5. a) Let  $f(x) = x^2 e^{-100x}$ . Over what interval is  $f(x)$  increasing? [5 points]

b) Let  $a$  be a positive constant. Find the largest value of  $(a + x)^3(a - x)^5$  as  $x$  ranges over the interval  $-a \leq x \leq a$ . [10 points]