淡江大學 95 學年度碩士班招生考試試題

110

系別：產業經濟學系　　科目：微　積　分

准帶項目請打「V」	
V	簡單型計算機

本試題共 1 頁

1. (a) (10%) Find the marginal-product functions for the CES production function
$$y = A \cdot [w_1 x_1^{-r} + w_2 x_2^{-r} + w_3 x_3^{-r}]^{-1/r},$$
where $A > 0$, $r > -1$, $0 < w_i < 1$ for $i = 1,2,3$ and $w_1 + w_2 + w_3 = 1$.

   (b) (10%) Compute all the first- and second-order derivatives of the function
$$f(x_1, x_2, x_3) = ax_1 + x_2^{\beta} x_3^{\gamma}$$
and show that Young's theorem applies.

   (c) (10%) Evaluate the following integrals: $\int 6x \cdot \exp(x^2) dx$ and $\int \frac{3x^2 + 2}{x^3 + 2x} dx$.

2. (15%) An important economic application of sequences is the determination of the present value of a sum of money to be received at some point in the future. The sequence $PV_t = V / [(1 + r)^t]$ represents the present value of an amount of money V received t periods into the future, where r is rate of interest. Suppose that a stream of equal payments of amount \$10,000 per year is to continues in perpetuity. At the interest rate of 6% compute

   (a) the present value of this entire stream of benefits.

   (b) the present value of the benefits beginning at the end of the 50^{th} year.

   (c) the present value of the first 50 years of benefits.

3. (15%) Economists often adopt (dis)continuous functions to represent economic relationships. Let Q and L denote output and labor, respectively. Given the production function $Q(L) = bL$, $b > 0$, defined on $[0, +\infty]$, derive

   (a) the cost function, $C(Q)$, and

   (b) the profit function, $\pi(Q)$ for a perfectly competitive firm. Let fixed costs be c_0, and let w be the unit price of L.

   (c) Prove that the cost function are continuous.

4. (15%) Given the following Walrasian model of a perfectly competitive market:
$$Q^D = 20 - 4P, \quad Q^S = -1 + 2P, \quad Q^D - Q^S = 0.$$

   (a) Solve for the equilibrium price, $\bar{P}$, and quantity transacted, $\bar{Q}$.

   (b) Show that the condition for Walrasian stability is satisfied.

   (c) If the market price initially were $P_1 = 4$, what type of disequilibrium would exist and how would the market adjust?

5. (10%) Solve the following constrained maximization and minimization problems.

   (a) max $y = x_1^{0.25} x_2^{0.75}$ subject to $2x_1^2 + 5x_2^2 = 10$.

   (b) min $y = 2x_1 + 4x_2$ subject to $x_1^{0.25} x_2^{0.75} = 10$.

6. (15%) Consider the following market equilibrium model. Price adjusts to excess demand as given by $\dot{p} = \alpha(q^D - q^S)$ and the number of firms in the industry adjusts to excess profits according to $\dot{N} = \gamma(p - \bar{c})$, where $Q^D = a + bp$ is the demand function and the supply function is $Q^S = (F + Gp)N$. Assume that $b < 0$ and all other parameters $(\alpha, \gamma, a, F, G, \bar{c})$ are greater than zero. Show that if γ is not too large, the equilibrium is a stable node, but that if γ is large enough, the steady state could be a stable focus.