淡江大學九十三學年度碩士班招生考試試題

系別：產業經濟學系　　　　科目：微　　積　　分

准帶項目請打「○」否則打「×」	
簡單型計算機	X

每題十分　詳列過程

1. Solve the differential equation $\dfrac{dy}{dt} + \dfrac{2y^4 t + 3t^2}{4y^3 t^2} = 0$

2. Let a firm's total cost function be $C = \alpha Q^3 + \beta Q^2 + \gamma Q + \delta$. If the marginal cost function is convex and positive for all output, what parameter restrictions are called for?

3. Suppose a wine grower owns a particular quantity of wine, which can be sold at present time $(t = 0)$ for price p_0 or stored for t years and then sold at a higher value. The growing value of the wine is a function of time $V = e^{\sqrt{2t}}$. Let the compound interest rate be r. Derive the optimal timing to maximize the present value of wine.

4. Let a firm's profit function be $\pi = pf(L,K) - wL - rK$, where p, w. r, L, and K stand for product price, wage rate, interest rate, labor and capital. Please find the equilibrium conditions for the profit-maximizing firm and give the sufficient condition. Then, do the comparative-static analysis for optimal L and K when wage rate changes.

5. Determine if the following function is concave or convex.

$$y = f(x_1, x_2, x_3) = x_1^a + x_2^b + x_3^c, \qquad 0 < a,b,c < 1,$$

6. Solve the constrained minimization problem and check the second order condition:
   $$\min \quad z = x + y \qquad s.t. \qquad x^{0.5} + y = 1$$

7. Find $\displaystyle\int \dfrac{x^3}{\sqrt{1+x^2}}\,dx$

8. Evaluate $\displaystyle\int (4e^x + 2x^2)(e^x + x)\,dx$

9. Find the derivatives of $y = \log_7 7x^2$ and $y = 6^{5x+3}$

10. Solve the differential equation: $2\dfrac{dy}{dt} + 4y = 6; y(0) = 1.5$