

# 淡江大學九十一學年度碩士班招生考試試題

系別：產業經濟學系

科目：微 積 分

准帶項目請打「○」否則打「×」	
計算機	字典
X	X

本試題共 / 頁

1. Determine whether the following series converge or diverge:

(1)  $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln(n+1)}$  (5%)    (2)  $\sum_{n=0}^{\infty} \frac{2^n}{n!}$  (5%)    (3)  $\sum_{n=1}^{\infty} \frac{4\sqrt{n}-1}{n^2+2\sqrt{n}}$  (5%)

2. Find (1)  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ , if  $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$ . (8%)

(2)  $\frac{\partial w}{\partial s}$  and  $\frac{\partial w}{\partial t}$ , if  $w = xy + yz + xz$ ,  $x = s \cos t$ ,  $y = s \sin t$ , and  $z = t$ ,  
when  $s = 1$  and  $t = 2\pi$ . (8%)

3. Find a power series centered at  $x = 1$  for  $f(x) = \ln x$ . (8%)

4. Evaluate (1)  $\int_0^{\sqrt{3}} x^3 \sqrt{4-x^2} dx$  (8%)

(2)  $\int_0^1 \frac{3x+4}{x^3-2x-4} dx$  (8%)

5. Find the local extrema of  $f(x, y) = x^3 - 4xy + 2y^2$ , if any exist. (10%)

6. For the CES (constant elasticity of substitution) production function

$$Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{1}{\rho}},$$

where  $K$  and  $L$  represent two factors of production, and  $A, \delta, \rho$  are three parameters  
( $A > 0$ ,  $0 < \delta < 1$ ,  $-1 < \rho \neq 0$ ), demonstrate

(1) the CES production function is quasiconcave for positive  $K$  and  $L$ . (10%)

(2) the CES function approaches the Cobb-Douglas function ( $AK^\delta L^{1-\delta}$ ) as  $\rho \rightarrow 0$ . (10%)

7. Investigate the scalar dynamical system

$$x_{t+1} = f(x_t) \text{ where } x_t \geq 0 \text{ for each } t, \text{ and}$$

$$f(x) = (1/2)(1+x) \quad \text{if } 0 \leq x < 2$$

$$= 2 + (1/2)x \quad \text{if } x > 2$$

Compute all steady states and examine their asymptotic stability. What is the asymptotic behavior of orbits starting in the interval  $(0, 2)$  and in the interval  $(2, \infty)$ ? (15%)