淡江大學八十八學年度碩士班招生考試試題

系別：產業經濟學系　　科目：微積分　　本試題共 / 頁

1. Let the function $f(x) = |x-1| + 1$ (15 %)
   ① Is this function continuous at $x=1$? why ?
   ② Is this function differentiable at $x=1$? why ?
   ③ sketch the graph of $f(x)$.

2. Given $z = 3y^{15}$, $y = x^2 + 5x - 1$, please find the derivative $\frac{dz}{dx}$ = ? (5 %)

3. Given $y = f(x) = 5x^5 + 3x^3 + x$ ① Does there exist an inverse function $x = f^{-1}(y)$? why?
   ② If there exists an inverse function $x = f^{-1}(y)$. find $\frac{dx}{dy}$ by inverse function rule. (10 %)

4. Given $y = f(x_1; x_2)$, find the total differential dy = ? and d^2y = ? (10%)

5. For the total cost function $TC(Q) = 3Q^2 + 7Q + 24$, $Q > 0$, show that marginal cost is less than average cost where AC is falling. (10 %)

6. Find the extreme values of $f(x,y) = 8x^3 + 2xy - 3x^2 + y^2 + 1$ and find out if the solution is relative maximum, relative minimum or saddle point ? (15 %)

7. Integrate the following ① $\int 8x^2(3x^3-1)^{16}\,dx$ ② $\int (e^{2x}x^3 + \frac{1}{x^3})\,dx$ (10 %)

8. Suppose a monopolist is practicing price discrimination in the sale of a product by charging different prices in two separate markets. Suppose that the demand curves are $P_1 = 80 - q_1$, $P_2 = 50 - q_2$ and the cost function is $C = 6(q_1 + q_2)$. How much should be sold in the two markets to maximize profits ? What are the prices charged ? How much profit is lost if price discrimination is made illegal ? (15 %)