

系別：企業管理學系

科目：微 積 分

准帶項目請打「V」

簡單型計算機

本試題共 | 頁

1. Evaluate the following limits, integrals or series: (36%)

(a) $\lim_{x \rightarrow 9} \frac{\sqrt{x}(\sqrt{x} - 3)}{x - 9}$

(b) $\lim_{h \rightarrow 0} \frac{\int_1^{1+h} \sqrt{5x+2} - \sqrt{10-x} \, dx}{h}$

(c) $\int_{-1}^1 x|x| \, dx$

(d) $\int_0^{\pi} e^x \sin x \, dx$

(e) $\sum_{x=0}^{\infty} \frac{\lambda^{2x}}{(2x)!}$, where $\lambda > 0$ is a constant.

(f) $\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$

2. Show that $2 < e < 3$. (10%)3. Suppose that $f(x)$ and $g(x)$ are such that $f^2(x)$ and $g^2(x)$ are integrable on $[a, b]$. Show that

$$\left[\int_a^b f(x)g(x) \, dx \right]^2 \leq \left[\int_a^b f^2(x) \, dx \right] \left[\int_a^b g^2(x) \, dx \right]. \quad (10\%)$$

4. Let $r(x)$ be the revenue from selling x items and $c(x)$ be the cost of producing the x items. Suppose that $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit? If so, what is it? (12%)5. Find the area of the region bounded by the curve $y = xe^{-x}$ and the x -axis from $x = 0$ to $x = 4$. (10%)6. Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$. (10%)

7. Solve the equation

$$x \frac{dy}{dx} = x^2 + 3y, \quad x > 0,$$

given the initial condition $y(1) = 2$. (12%)