淡江大學九十四學年度碩士班招生考試試題 138-1

系別：企業管理學系　　科目：微　積　分

准帶項目請打「V」	
	簡單型計算機

本試題共 一 頁

注意事項：需列出計算過程，否則不予計分。

1. Find the following integrals.

   a. $\int_0^{0.5} \frac{x^3+7x^2-5x+5}{(x-1)^2(x+1)^3}\,dx$ (6%)

   b. $\int_0^{\pi} e^{2x}\cos x\,dx$ (6%)

2. Evaluate each limit if they converge.

   a. $\lim_{x\to 0^+}(\frac{1}{x})^{\sin x}$ (6%)

   b. $\lim_{x\to y}\frac{x^n-y^n}{x-y}$ (6%)

   c. $\lim_{n\to\infty}\sqrt[8]{n^2+1}-\sqrt[4]{n+1}$ (6%)

3. Decide whether each of the following infinite series is convergent or divergent.

   a. $\sum_{n=2}^{\infty}\frac{1}{n\log n}$ (7%)

   b. $\sum_{n=1}^{\infty}\frac{2^n n!}{n^n}$ (7%)

4. Find $F'(x)$, if

   a. $F(x)=\int_a^{x^3}\sin^2 t\,dt$ (5%)

   b. $F(x)=\int_0^x xf(t)\,dt$. (5%)

   c. $F(x)=[x]$, where $[\cdot]$ denotes the Guess function. (6%)

5. Find $f'(0)$, if $f(x)=\begin{cases} g(x)\cos x, & x\neq 0 \\ 0, & x=0 \end{cases}$ and $g(0)=0,\ g'(0)=2$. (10%)

6. Suppose that a tank initially contains 10 gallons of pure water. Brine containing 3 ponds of salt per gallon flows into the tank at a rate of 2 gallons per minute, and well-stirred mixture flows out of the tank at the same rate. How much salt is present at the end of 10 minutes? How much salt is present in the long run? (15%)

7. Find the maximum and minimum values of $f(x,y,z)=x-2y+z$, with the constraints $x^2+y^2+z^2=1$ and $x+y+z=0$. (15%)