淡江大學98學年度碩士班招生考試試題

系別：電機工程學系　　　　　　　科目：電　　磁　　學

准帶項目請打「V」	
✔	簡單型計算機

本試題共 2 頁，1 大題

15%　1a)　For spherical coordinates, the unit base vectors that describe field components change in different directions depending on the observation point. If the observation points are located at $\theta = 0$ and $\phi = 0$, what are $\hat{a}_r$, $\hat{a}_\theta$, and $\hat{a}_\phi$ in terms of Cartesian unit base vectors?

1b) Explain why the divergence of a vector $\vec{D}$ in spherical coordinates can not be evaluated by the following straight-summation of the partial derivatives of each component:

$$\textbf{Wrong:}\ \nabla \cdot \vec{D} = \frac{\partial D_r}{\partial r} + \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_\theta}{\partial \theta}$$

20%　2)　A biconical surface is defined by the regions $\theta = 30°$, $\theta = 150°$, and $r < 2$m. The upper cone has a positive, uniform surface charge density of $+\rho_s$, and the lower cone has a negative, uniform surface charge density of $-\rho_s$. Please derive an expression to calculate the electrostatic field E for any point (x,y,0) on the xy plane due to these charge distributions. ***<u>You should simplify your expression as much as possible, but don't try to evaluate the final integral(s).</u>***

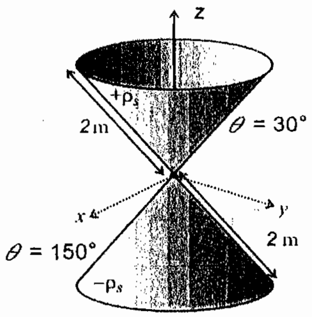


Fig. 2

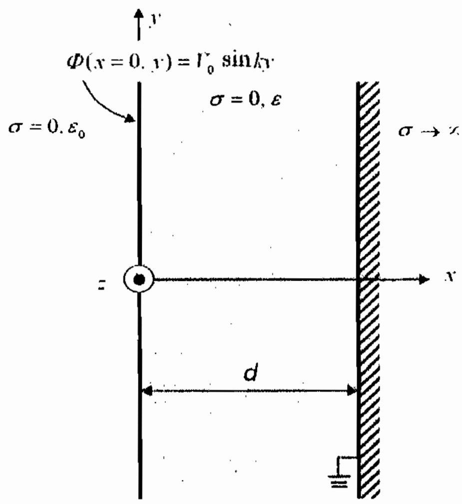


Fig. 3

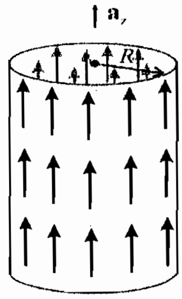
25%　3)　A potential sheet of infinite extent in the y and z directions is placed at $x = 0$ and has potential distribution $\Phi(x=0,y)=V_0\sin ky$. Free space with no conductivity ($\sigma = 0$) and permittivity ε_0 is present for $x < 0$, while for $0 < x < d$ a perfectly insulating dielectric ($\sigma = 0$) with permittivity ε is present. The region for $x > d$ is a grounded perfect conductor at zero potential.

a) Solve the Laplacian equation for the potential distributions $\Phi(x,y)$ for $x < 0$ and $0 < x < d$?

b) What are the surface charge densities at $x = 0$, (i.e., ρ_s (x= 0, y)), and at $x = d$ (i.e., ρ_s (x=d ,y)) ?

20%　4) Rather than view a current in a wire as a line current, it is more realistic to treat it as a volume charge flowing over a cylindrical area with the following current density:

$$\vec{J} = \begin{cases} J_0 \hat{a}_z & \text{for } \rho \le R \\ 0 & \text{for } \rho > R \end{cases}$$



section of infinite volume current density

Answer the following questions:

(a) Use Ampere's law to derive a solution for H at all points in space.

(b) If the conductivity of the wire is σ, write an expression for the E field inside the wire.

(c) Prove that your answer in part (a) satisfies Maxwell's curl equation.

Hint: the curl of a vector in cylindrical coordinates is

$$\nabla \times \vec{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\hat{a}_\phi + \frac{1}{\rho}\left(\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right)\hat{a}_z$$

20%　5a) Below are the solutions to a transmission line partial differential equation:

$$v(z,t) = 80\cos(2000t - 10z) - 160\pi u(4t + z/50 + 14)\text{V}$$

$$i(z,t) = 2\pi u(4t + z/50 + 14) + \cos(2000t - 10z)\text{A}$$

Answering the following questions about these solutions:

i) Write the forward-propagating voltage waveform !

ii) Write the backward-propagating current waveform !

iii) What is the characteristic impedance for this line ?

iv) What is the velocity of propagation for this line ?

5b) If a lossless transmission line (with charteristic impedance Z_0) were connected to a purely resistive load R_L and had a VSWR of 2 (linear scale), what are the two possible values for the load resistance in terms of Z_0?