淡江大學九十二學年度碩士班招生考試試題

系別：電機工程學系　　科目：電　磁　學(含電磁波)

准帶項目請打「○」否則打「×」
簡單型計算機　×

1. (20%) A thin wire is bent into a large horizontal circular ring of radius a and uniformly charged to total charge $-q$. A point charge of mass m and charge $+q$ is placed at the exact center of the ring. If this point charge is displaced precisely vertically, but only slightly ($z \ll a$), from the center and released, it will oscillate about the center point.
   (a) Find the electric field $\vec{E}$ at the point charge for $z \ll a$.
   (b) Verify that the ensuing motion is that of a simple harmonic oscillator and find the frequency ω of the oscillation.

2. (20%) A charge q revolves in the x-y plane around z axis, along a circular orbit of radius a, at constant angular velocity ω.
   (a) Use Biot-Savart's law to find the magnetic field $\vec{H}(z,t)$ along the z-axis. (Note : $I\vec{dl} = q\vec{u}$, $\phi = \omega t$)
   (b) Find the time average of this magnetic field $\langle \vec{H}(z,t) \rangle$ (meaning the time-integral of $\vec{H}$ over one period, divided by that period) and compare it with $\vec{H}$ on the z axis of a circular loop of current I in the x-y plane, of radius a.

3. (20%) Two large parallel circular conducting plates are separated by a nonconducting dielectric with permittivity ϵ and permeability μ. The plates are close enough such that the fringing is neglected. Suppose that an electric field is given to be $\vec{E} = E_0 \sin \omega t \, \vec{a}_z$ between the plates.

◀注意背面尚有試題▶

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(a) From evaluating $\oint_C \vec{H}\cdot d\vec{\ell}$, where C is a horizontal circle of typical radius ρ, centered on the z axis, inside the plates, find $\vec{H}(\rho)$.

(b) From Maxwell's equation $\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$, what does this say about the time-dependence of $\vec{H}$? What does the answer of part (a) say about the time-dependence of $\vec{H}$?

(c) Discuss the inconsistency in part (b). What is wrong with the given $\vec{E}$?

4. (20%) A uniform plane wave having the electric field

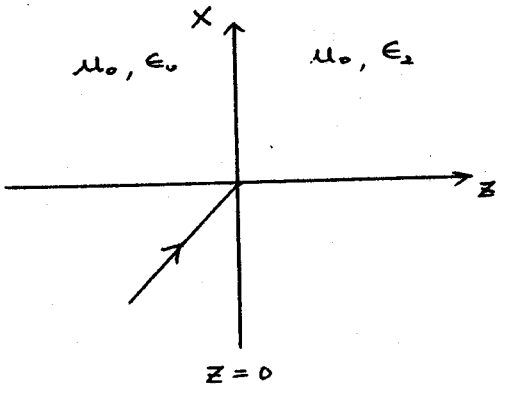
$$\vec{E}_i(t) = E_0\left(\frac{1}{2}\vec{a}_x - \frac{\sqrt{3}}{2}\vec{a}_z\right)\cos\left[\omega t - 10\pi(\sqrt{3}x+z)\right] + \vec{a}_y E_0 \sin\left[\omega t - 10\pi(\sqrt{3}x+z)\right]$$

is incident on the interface between free space and a dielectric medium of unknown ϵ_2 and $\mu_2 = \mu_0$.

(a) Find the frequency and wavelength of the incident wave.

(b) Find the value of ϵ_2 for which the reflected wave is linearly polarized.

(c) For the found value of ϵ_2 in (b), find the expressions of $\vec{E}(t)$ for the reflected and the transmitted waves.



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5. (a) From Maxwell's equations, derive the wave equation for the vector magnetic potential $\vec{A}$, (20%)

$$\vec{\nabla}^2\vec{A} - \mu\epsilon\frac{\partial^2\vec{A}}{\partial t^2} = -\mu\vec{J},$$

under the Lorentz gauge condition $\vec{\nabla}\cdot\vec{A} = -\mu\epsilon\frac{\partial V}{\partial t}$.

(b) Describe the reasoning (not the proof) that the solution of the above equation is

$$\vec{A}(\vec{r},t) = \int_{v'}\frac{\mu\vec{J}(\vec{r}',t-\frac{R}{u})}{4\pi R}dv', \quad \text{where } R=|\vec{r}-\vec{r}'|,\ u=\frac{1}{\sqrt{\mu\epsilon}}.$$

compared with the static case.

(c) For a current along the z axis in a practical antenna, show that $\mu H_\phi = -\sin\theta\frac{\partial A_z}{\partial r}$, when only the distant field ($\frac{1}{r}$ term) is considered.