淡江大學 104 學年度碩士班招生考試試題 29

系別：電機工程學系通訊與電波組　科目：工程數學

考試日期：3 月 8 日(星期日) 第 2 節　本試題共 8 大題， 1 頁

1. (10%) Find the derivative

$$\frac{d}{dx}\ln\frac{x+1}{\sqrt{x-1}}.$$

2. (10%) Find the integral

$$\int_0^1 \sqrt{1-x^2}\,dx.$$

3. (10%) Given the vector function $\mathbf{F}(x,y,z) = y^2\mathbf{i} + \left(2xy + e^{3z}\right)\mathbf{j} + 3ye^{3z}\mathbf{k}$, find the potential function $f(x,y,z)$ such that $\nabla f = \mathbf{F}$.

4. (10%) Assume that C is the line segment from $(-5,-3)$ to $(0,2)$, find the line integral

$$\int_C \left(y^2dx + xdy\right).$$

5. (10%) Solve the initial value problem

$$e^y\frac{dy}{dx} - x = 0, \quad y(1) = 0.$$

6. (10%) Find the inverse Laplace transform of

$$F(s) = \frac{1}{(s+1)^2}, \quad s > -1.$$

7. Let X be a continuous random variable with probability density function (PDF)

$$f_X(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) (5%) Find the conditional probability $P(X \ge 0.4 \,|\, X \le 0.8)$.

(b) (5%) Find the expectation $\mathbb{E}[X]$.

(c) (5%) Find the cumulative distribution function (CDF) $F_X(x)$ of the random variable X.

(d) (5%) Let $Y = -\ln X$, find the PDF of random variable Y.

8. Given the matrix $\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$, answer the following questions.

(a) (5%) Find the eigenvalues of matrix $\mathbf{A}$.

(b) (5%) Find the matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix.

(c) (5%) Find the inverse matrix $\mathbf{A}^{-1}$.

(d) (5%) Find the eigenvalues of matrix $\mathbf{A}^{-10}$.