淡江大學 101 學年度碩士班招生考試試題

系別：電機工程學系通訊與電波組　科目：工　程　數　學

考試日期：2 月 26 日(星期日) 第 2 節　　本試題共　10　大題，　1　頁

1. (10%) Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$.

2. (10%) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -2 & 1 \\ 3 & 6 & -5 & 4 \\ 1 & 2 & 0 & 3 \end{bmatrix}$.

3. (10%) Find the orthogonal projection of the vector $\mathbf{u} = [1\ 2]$ onto the vector $\mathbf{v} = [1\ 1]$.

4. (10%) Find the inverse Laplace transform of

$$H(s) = \frac{1}{(s+2)(s+3)}.$$

5. (10%) Find the solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3 = 0,\ \frac{dx(0)}{dt} = 0,\ x(0) = 2.$$

6. (10%) Find the Fourier transform of the function

$$f(t) = \begin{cases} 2, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

7. (10%) Find the value of the integral

$$\int_0^1 xe^{-x}dx.$$

8. (10%) The events A_1 and A_2 partition the sample space. Let B be another event. Suppose the probabilities are

$$P(A_1) = 0.6,\ \ P(A_2) = 0.4,\ \ P(B|A_1) = 0.4,\ \ P(B|A_2) = 0.6.$$

Find the conditional probability $P(A_1|B)$.

9. (10%) Let X be uniformly distributed in the interval [0, 1]. Consider the random variable $Y = g(X)$, where

$$g(x) = \begin{cases} 1 & \text{if } x \le 1/3 \\ 2 & \text{if } x > 1/3 \end{cases}$$

Find the probability mass function (PMF) of the random variable Y.

10. (10%) Let X and Y be two independent standard Gaussian random variables. Find the mean and variance of the random variable $Z = XY$.