(1) Solve the equation

$$e^x \sin(y) - 2x + (e^x \cos(y) + 1)y' = 0 \quad (20\%)$$

(2) Solve the equation

$$y'' + 2y' - 3y = 8e^x, \; y(0)=4, \; y'(0)=-6 \quad (20\%)$$

(3) Compute the complex Fourier integral representation of $f(x)$. (20%)

$$f(x) = \begin{cases} e^{-ax} & \text{for } x \geq 0 \\ e^{ax} & \text{for } x < 0 \end{cases} \text{, where } a>0.$$

(4) Solve the problem

$$\frac{\partial^2 y}{\partial t^2} = 9\frac{\partial^2 y}{\partial x^2} \text{ for } -\infty<x<\infty, \; t \geq 0,$$

$$y(x,0) = 4e^{-5|x|} \text{ for } -\infty<x<\infty, \qquad (20\%)$$

$$\frac{\partial x}{\partial t}(x,0) = 0.$$

(5) (i) Find the Laurent expansion of $\cos(z)/z^5$ about zero. (10%)

(ii) Evaluate $\oint_\Gamma \frac{\cos z}{z^5}$, where Γ is a closed path enclosing zero. (10%)