

系別：電機工程學系

科目：工程數學

准帶項目請打「V」	
✓	簡單型計算機

本試題共 1 頁，5 大題

1. (a) (10%) Find the solution of the equation

$$\frac{dy}{dx} = \frac{2y - x}{2x - y}$$

- (b) (10%) Find the solution of the equation

$$\frac{dy}{dx} = \frac{2y - x + 5}{2x - y - 4}$$

Hint: To reduce the equation of part (b) to that of part (a), consider a preliminary substitution of the form $x = X - h$ and $y = Y - k$. Choose the constants h and k so that the equation is homogeneous in the variables X and Y .

2. (a) (10%) Verify that x and xe^x are solutions of the homogeneous equation corresponding to

$$x^2 y'' - x(x+2)y' + (x+2)y = 2x^3, \quad x > 0.$$

- (b) (10%) Find the general solution of the above differential equation.

3. (a) (10%) Let C be the circle $|z| = 2$, described in the positive sense. Evaluate the integral

$$\int_C \tan z dz.$$

- (b) (10%) Use the residual theorem to compute the integral

$$\int_{-\infty}^{\infty} \frac{\cos 3x}{(x^2 + 1)^2} dx.$$

4. It is known that the matrix A and B are row equivalent (that means matrix B can be obtained from matrix A by row operation).

$$A = \begin{bmatrix} 2 & -4 & 0 & 1 & 7 & 11 \\ 1 & -2 & -1 & 1 & 9 & 12 \\ -1 & 2 & 1 & 3 & -5 & 16 \\ 4 & -8 & 1 & -1 & 6 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -2 & 0 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & -5 & -3 \\ 0 & 0 & 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) (5%) Determine the rank of matrix A .
 (b) (5%) Determine the nullity of matrix A .
 (c) (5%) Find the basis of the null space of matrix A .
 (d) (5%) Find the basis of the row space of matrix A .

5. (a) (10%) Find the Fourier transform of the function

$$f(t) = \exp\left(-\frac{t^2}{2}\right), \quad -\infty < t < \infty.$$

- (b) (10%) Consider the periodic function $f(x + 2\pi) = f(x)$ for $-\infty < x < \infty$, where

$$f(x) = x, \quad -\pi < x < \pi.$$

Find the Fourier series expansion of $f(x)$, $-\infty < x < \infty$.