

1. Find the general solution of $y'' - \left(\frac{3}{x}\right)y' + \left(\frac{4}{x^2}\right)y = 0$, given that one solution is $y_1(x) = x^2$. (20%)

2. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, $(x > 0, y > 0)$,
 $u(0, y) = 0$ ($y > 0$), $u(x, 0) = \begin{cases} 4, & 0 \leq x \leq 2 \\ 0, & x > 2 \end{cases}$ (20%)

3. $\oint_{\Gamma} \frac{1+z^2}{(z-1)^2(z+2i)} dz$, Γ the circle of radius 7 about $-i$. (20%)

4. (a). determine the Fourier transform of the function (10%)

$$f(t) = \frac{5e^{3it}}{t^2 - 4t + 13}$$

(b). find the inverse Fourier transform of the function (10%)

$$F(\omega) = e^{-3|\omega+4|} \cos(2\omega+8)$$

5. Use the Laplace transform to solve the system (20%)

$$\begin{cases} x' + 2x - y' = 0 \\ x' + x + y = t^2 \end{cases}, \quad x(0) = y(0) = 0$$