

淡江大學九十四學年度碩士班招生考試試題 ¹⁰⁷⁻¹

系別：資訊工程學系

科目：邏輯導論與機率論

准帶項目請打「V」	
X	簡單型計算機
本試題共 / 頁	

1. Random variables X and Y have a joint PMF (probability mass function) given by the following matrix.

$P_{X,Y}(x, y)$	$y = -1$	$y = 0$	$y = 1$
$x = -1$	0	0.25	0.25
$x = 1$	0.25	0.25	0

Are X and Y independent? Are X and Y uncorrelated? (15%)

2. Random variables x and Y have the joint PMF (probability mass function) (10%)

$$P_{X,Y}(x, y) = \begin{cases} c|x+y| & x = -2, 0, 2; \\ & y = -1, 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What is the value of the constant c?
- (b) What is $P[Y > X]$?

3. Suppose that X is uniformly distributed over $[-1, 3]$ and $Y = X^2$. Find CDF (Cumulative Distribution Function) $F_Y(y)$ and the PDF (probability density function) $f_Y(y)$. (15%)

4. For independent events A and B, please prove that A and B' are independent, where X' denotes the complement of a set X. (10%)

5. Determine the truth value of each of these statements and then prove your answer. (20%)

- (a) $\exists x \in Z \forall y \in Z (x \leq y^2)$, where Z denotes the set of all integers.
- (b) $(p \rightarrow r) \vee (q \rightarrow r)$ is logically equivalent to $((p \wedge q) \rightarrow r)$.

6. What are the truth values of these statements? (15%)

- (a) $\exists! x P(x) \rightarrow \exists x P(x)$.
- (b) $\forall x P(x) \rightarrow \exists! x P(x)$.
- (c) $\exists! x \neg P(x) \rightarrow \neg \forall x P(x)$

7. Construct the given half adder using NAND gates. (15%)

