淡江大學九十二學年度碩士班招生考試試題

系別：資訊工程學系　　　　　科目：邏輯導論與機率論

1. (T or F, if "F" give your reason to get full credits.) (15 %)
   (a) $(p \to q) \to r \Leftrightarrow p \to (q \to r)$.
   (b) $\forall m \exists n (n > 2^m)$, $m,n \in \{ 0, 1, 2, 3, \ldots \}$.
   (c) $\exists n \forall m (n > 2^m)$, $m,n \in \{ 0, 1, 2, 3, \ldots \}$.

2. Write the negation of $\exists x \forall y \exists z [(z > y) \to (z < x^2)]$ without using the connective $\neg$. (10%)

3. Answer the following: (5 + 20 %)
   (a) For n=3: let $X_3 = \{1, 2, 3\}$. Now consider the sum

   $$S_3 = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3} = \sum_{A \subseteq X_3} \frac{1}{P_A}, \quad \text{where } A \neq \varnothing, \text{ and}$$

   P_A denotes the product of all elements in a nonempty subset A of X_3. Note that the sum is taken over all the nonempty subsets of X_3. Evaluate this sum S_3.

   (b) Conjecture the general result of S_n for $n \geq 1$. Prove your conjecture by mathematical induction.

   (hint: write and evaluate S_2 and S_4 to find the relations among S_k and S_{k+1}.)

4. If the probability table for events A, B, C, D, E is given in the follow, where {A, B} and {C, D, E} are partitions of the sample space respectively. If we know A & D are independent and A, C are disjoint, and $P[A \cap D] = 1/12$, $P[A] = 1/4$, $P[C|B] = 1/8$, find the values for ① ② ③. Show all your work to get full credits. (15 %)

P[•]	C	D	E
A		$\frac{1}{12}$	②
B	①		③

5. Random variables X & Y have the joint pmf (probability mass function)

   $$P_{X,Y} = \begin{cases} \frac{1}{21} & x = 0,1,2,3,4,5; \ y = 0,1,\ldots,x, \\ \\ 0 & \text{otherwise.} \end{cases}$$

   Find the marginal pmf of X and Y respectively, also find the covariance Cov[X,Y]. (20%)

6. Let X and Y be independent continuous random variables and uniformly distributed over (0, 1). Find the density of X+Y. (15%)