淡江大學九十一學年度碩士班招生考試試題

系別：資訊工程學系　　科目：邏輯導論與機率論　　97-1

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本試題雙面印製

Part 1

1. Define the two operators NAND (not AND) and NOR (not OR) as follows

   α NAND $\beta \equiv \neg(\alpha \wedge \beta)$

   α NOR $\beta \equiv \neg(\alpha \vee \beta)$

   Obtain the truth tables for NAND and NOR (5%)

2. Use the predicates

   P(x) : x is a peacock

   T(x) : x is proud of its tail

   S(x) : x can sing

   (A). Give the corresponding First Order Logic well-formed formulas (wffs) for the following statements (assume the universe is the set of all birds)

   (1). No birds, except peacocks, are proud of their tails (5%)

   (2). Some birds that are proud of their tails cannot sing. (5%)

   (3). Some peacock cannot sign. (5%)

   (B). Show that statement (1) is a conclusion of (2) and (3). (10%)

3. Determine which of the following are *satisfiable*: (10%)

   (a) $(P \vee Q \vee R)(\neg P \vee \neg Q \vee \neg R)(\neg P \vee Q)(\neg Q \vee R)$

   (b) $(P \wedge Q) \vee R \Rightarrow (\neg Q \vee \neg R)$

   (c) $\neg(P \wedge Q) \Rightarrow \neg P \vee Q$

   (d) $\neg(P \vee Q) \Rightarrow \neg P \vee Q$

   (e) $P \wedge Q \Rightarrow \neg Q$

4. Prove or disprove the following are *tautologies*:

   (a) $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \wedge \neg R) \Rightarrow \neg Q)$ (5%)

   (b) $\neg(P \wedge Q) \Leftrightarrow \neg P \vee Q$ (5%)

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Part 2

1. Let X be a random variable whose distribution function F is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ x/2 & 0 \le x < 2 \\ x/3 & 2 \le x < 3 \\ 1 & x \ge 2 \end{cases}$$

Find

(a) $P(1/2 \le X \le 1)$ (2%)

(b) $P(1/2 \le X \le 5/2)$ (2%)

(c) $P(1 \le X \le 3/2)$ (2%)

(d) $P(1/2 \le X < 5)$ (2%)

(e) $P(1 < X < 3)$ (2%)

2. Let $X_1, \ldots, X_n$ be independent random variables having common density function with mean μ and variance σ^2. Set $\overline{X} = (X_1 + X_2 + \ldots + X_n)/n$. Prove

(a) $\sum_{k=1}^{n}(X_k - \overline{X})^2 = \sum_{k=1}^{n}(X_k - \mu)^2 - n(\overline{X} - \mu)^2$ (15%)

(hint : consider $X_k - \overline{X} = (X_k - \mu) - (\overline{X} - \mu)$)

(b) $E(\sum_{k=1}^{n}(X_k - \overline{X})^2) = (n-1)\sigma^2$ (10%)

(hint : use result of part (a))

3. Let X be a continuous random variable with density function f given by

$$f(x) = (1/2)\, e^{-|x|} \qquad -\infty < x < \infty$$

Find $P(1 \le |X| \le 2)$ (15%)