淡江大學九十學年度碩士班招生考試試題

系別：資訊工程學系　　科目：邏輯導論與機率論

准帶項目請打「○」否則打「×」	
計算機	字典

本試題雙面印製

(1) (a) If p, q, r, t are primitive statements, please **prove** that
$(p \vee q \vee r) \wedge (p \vee t \vee \neg q) \wedge (p \vee \neg t \vee r) \Leftrightarrow p \vee [r \wedge (t \vee \neg q)]$,
by the laws of logic, and **give reason(s) for each step**.

(b) Write **the dual** of the logical equivalence in (a).

(2) **Prove** the following result **in three ways**,
"If n is an odd integer, then n+7 is even".

(3) Assume the number of people enter a post office distributed as a Poisson distribution, with mean 90 / ***hour*** .
We also know that
$P($ ***person enters the post office is a male*** $) = p$
$P($ ***person enters the post office is a female*** $) = 1 - p$
Let X = number of males enter the post office,
Y = number of females enter the post office.
Please **find**
(a) the **joint** probability density function of X and Y,

(b) the **marginal** probability density functions of X and Y, respectively.

(4) Random variable X is distributed as $Uniform(-1,5)$.
Random variable Y is distributed as $Uniform(3,9)$.

(a) If $W = |X|$, please

(i) **find** the probability density function for W,
and (ii) **prove** that the answer in (i) **is** a probability density function.

(b) Please **find** the probability density functions of $X+Y$.

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(5) A CPU serves 2 incoming queues of jobs.

Let X = number of jobs serviced from queue A, before the head job of queue B is serviced.

$P(\textit{selecting queue } A) = p$

$P(\textit{selecting queue } B) = 1 - p$

The choice (of which queue to service) is random, and each selection of job is independent of the preceding selection.

Let A_k =selection k is from queue A.

Let B_k =selection k is from queue B, k=1,2, … .

(a) Please (i) **find** the probability density function for X, and (ii) **prove** that the answer in (i) is a probability density function.

(b) Now, assume that an Operating System is designed to give preference to "long-waiting jobs" with selection rules as follows:

$$P(A_1) = p, \quad P(A_2 \mid A_1) = \frac{p}{2}, \quad P(A_3 \mid A_1 \cap A_2) = \frac{p/2}{2}, \quad \cdots\cdots,$$

$$P(B_1) = 1 - p, \quad P(B_2 \mid B_1) = \frac{1-p}{2}, \quad P(B_3 \mid B_1 \cap B_2) = \frac{(1-p)/2}{2}, \quad \cdots\cdots.$$

Please (i) **find** the probability $P(X = 2)$, and (ii) **find** the probability density function for X.