

## 淡江大學八十九學年度碩士班招生考試試題

系列：資訊工程學系

科目：邏輯導論與機率論

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## Part 1

1. Prove that the following are tautologies. (20%)
  - (a)  $[(p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)] \rightarrow (\neg q \rightarrow s)$
  - (b)  $[(p \rightarrow q) \wedge (r \rightarrow s) \wedge (\neg q \vee \neg s)] \rightarrow (\neg p \vee \neg r)$
2. Let the universe for the variables in the following statements consist of all real numbers. In each case negate and simplify the given statement (20%)
  - (a)  $\forall x \forall y [(x < y) \rightarrow \exists z (x < z < y)]$
  - (b)  $[\forall x \forall y ((x > 0) \wedge (y > 0))] \rightarrow [\exists z (xz > y)]$
3. Prove that for any integer  $n$ ,  $n^2$  is even if and only if  $n$  is even. (10%)

## Part 2

4. Show that for all  $n \in \mathbb{Z}^+$  (15%)

$$\binom{2n}{n} = \sum_{\lambda=0}^n \binom{n}{\lambda}^2$$

5. Let  $X$  be geometrically distributed with parameter  $p$ . Compute the density of (20%)
  - (a)  $X^2$
  - (b)  $X + 5$
6.  $X$  has the gamma density  $\Gamma(\alpha, \lambda)$ , find  $EX$ . (15%)