

淡江大學八十八學年度碩士班招生考試試題

系別：資訊工程學系

科目：邏輯導論與機率

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本試題雙面印製

Part I

1. Prove the following are tautologies:

(a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$ (10%)

(b) $\{[p \rightarrow (q \vee r)] \wedge (\sim q)\} \rightarrow (p \rightarrow r)$. (10%)

2. Verify that the following argument is valid by using the rules of inference:

If Clifton does not live in France, then he does not speak French.

Clifton does not drive a Datsun.

If Clifton lives in France, then he rides a bicycle.

Either Clifton speaks French, or he drives a Datsun.

Hence, Clifton rides a bicycle. (10%)

3.

(a) Prove by contrapositive: If a is an odd integer, then there are no integral roots for the polynomial $F(X) = X^2 - X - a$. (10%)

(b) Prove by contradiction: There do not exist 3 consecutive integers such that the cube of the largest is equal to the sum of the cubes of the 2 other integers. (10%)

◀ 注意背面尚有試題 ▶

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Area $\Phi(x)$ under the Standard Normal Curve to the Left of x

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879

Part II

1. Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that $X = 20$. Use the normal approximation and then compare it to the exact solution. (10%)

2. Suppose an urn has r red balls and b black balls. A ball is drawn and its color noted. Then it together with $c > 0$ balls of the same color as the drawn ball are added to the urn. The procedure is repeated $n-1$ additional times so that the total number of drawings made from the urn is n .

Let $R_j, 1 \leq j \leq n$, denote the event that the j th ball drawn is red and let $B_j, 1 \leq j \leq n$, denote the event that the j th ball drawn is black. Find the probability that

(a) the second ball is red. (10%)

Given that the second ball was red, find the probability that

(b) the first ball was red. (10%)

3. Suppose that whether or not it rains today depends on previous weather conditions the last two days. That is, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday? (10%)

4. Suppose that X_1 and X_2 are independent exponential random variables with respective means $1/\lambda_1$ and $1/\lambda_2$; then what is $P\{X_1 < X_2\}$? (10%)