淡江大學八十七學年度碩士班入學考試試題

系別： 資訊工程學系　　　　科目： 邏輯導論與機率論

Show enough work to justify your answer!!

Part A: Logic

Notation for logical connectives:

$\wedge$ for *and*, $\vee$ for *or*, $\neg$ for *not*, $\rightarrow$ for *imply*,

$\Rightarrow$ for *logical imply*, $\Leftrightarrow$ for *logical equivalent*

1. Establish the validity of the following argument. (10 points)

$$[(p \rightarrow q) \wedge [(q \wedge r) \rightarrow s] \wedge r] \rightarrow (p \rightarrow s)$$

2. Prove or disprove:

   (a) $\exists x \, [p(x) \wedge q(x)] \Leftrightarrow [\exists x \, p(x) \wedge \exists x \, q(x)]$ (5 points)

   (b) $\forall x \, [p(x) \wedge q(x)] \Leftrightarrow [\forall x \, p(x) \wedge \forall x \, q(x)]$ (5 points)

3. Here are two sentences (A) and (B):

   (A): $\forall x \, \exists y \, p(x,y)$

   (B): $\exists y \, \forall x \, p(x,y)$

   Give an interpretation, and show that (A) follows from (B), i.e. (B) $\Rightarrow$ (A). (10 points)

4. Formulate the following sentences into well-formed formula.

   The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of American, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

   Answer the query "Is West a criminal" by resolution techniques. (20 points)

   You can apply the resolution technique below.

   $$[(p \vee q) \wedge (\neg q \vee r)] \Rightarrow (p \vee r)$$

Part B: Probability

1. If the letters in the acronym WYSIWYG are arranged in a random manner, what is the probability the arrangement starts and ends with the same letter. (10 points)

2. Two gamblers, A and B, bet on the outcomes of successive flips of a coin. On each flip, if the coin comes up head, A collects from B one unit, whereas, if it comes up tails, A pays to B one unit. They continue to do this until one of them runs out of money. If it is assumed that successive flips of the coin are independent and each flip results in a head with probability p, what is the probability that A ends up with all the money if he starts with i units and B starts with $N - i$ units. (15 points)

3. Consider a circle of radius R and suppose that a point within the circle is randomly chosen in such a manner that all regions within the circle of equal area are equally likely to contain the points. (In other words, the point is uniformly distributed within the circle.) If we let the center of the circle denote the origin and define X and Y to be the coordinates of the point chosen, it follows that, since (X,Y) is equally likely to be near each point in the circle, that the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} c & if\ x^2 + y^2 \leq R^2 \\ 0 & if\ x^2 + y^2 > R^2 \end{cases}$$

for some value of c.

(a) Determine c. (7 points)

(b) Compute the probability that the distance from the origin of the point selected is not greater than a. (8 points)

4. A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that will take him to safety after 3 hours of travel. The second door leads to a tunnel that will return him to the mine after 5 hours of travel. The third door leads to a tunnel that will return him to the mine after 7 hours. If we assume that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until he reaches safety? (10 points)