

淡江大學 99 學年度轉學生招生考試試題

系別：理工組二年級

科目：微 積 分

本試題共

大題，> 頁

第一部份 簡答題 (60%)，(不需寫出演算過程，答案依照題號寫在答案卷第一頁，題號要清楚標明，共十題，每小題六分)：

1. Find $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - \sqrt{x^2 - x})$.
2. Find an equation for the tangent to the curve $y = x^3 - 4x + 1$ at the point $(2, 1)$.
3. Find $\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \frac{t^2}{t^4 + 1} dt$.
4. Evaluate the improper integral $\int_{-1}^{\infty} \frac{dx}{x^2 + 5x + 6}$.
5. The region bounded by the curve $y = \sqrt{x}$, the x -axis, and the line $x = 4$ is revolved(迴轉) about the x -axis to generate a solid. Find the volume(體積) of the solid.
6. Find the length of the curve $r = \sqrt{1 + \cos 2\theta}$, $0 \leq \theta \leq \pi\sqrt{2}$.
7. Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n^2 + 3}}$.
8. Find the derivative of $f(x, y) = xe^y + \cos xy$ at the point $(2, 0)$ in the direction $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$.
9. Calculus the iterative integral $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$.
10. Calculus the iterative integral $\int_0^{\ln 2} \int_0^{\sqrt{(\ln 2)^2 - y^2}} e^{\sqrt{x^2 + y^2}} dx dy$.

背面尚有試題

第二部份 計算證明題 (40%)，(演算過程必須寫清楚，直接寫答案不計分，共四題，每小題十分)：

1. Show that for any real number a and b , the inequality $|\sin b - \sin a| \leq |b - a|$ is true.
2. Sketch the curve(畫詳圖) $y = \frac{x}{x^2 + 1}$.
3. Find the maximum and minimum values of the function $f(x, y) = x + 2y$ subject to the constraints $x + y + z = 1$ and $y^2 + z^2 = 4$.
4. Find the volume(體積) of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.