淡江大學 98 學年度轉學生招生考試試題

系別：理工組二年級　　科目：微　積　分

准帶項目請打「V」	
	計 算 機

本試題共 2 大題，　頁

第一部份 簡答題（60 %），（不需寫出演算過程，答案依照題號寫在答案卷第一頁，題號要清楚標明，共十題，每小題六分）：

1. Evaluate $\lim_{x\to 2} \dfrac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$.

2. Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$$

3. If $F(x) = \int_1^x f(t)\,dt$, where $f(t) = \int_1^{t^2} \dfrac{\sqrt{1+u^2}}{u}\,du$, find $F''(2)$.

4. If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

5. Find y' if $y = x^{\sin x}$.

6. Evaluate the integral $\int e^{2x} \sin 3x\,dx$.

7. Find $\partial z/\partial s$ where $z = e^x \cos y$, $x = st$, and $y = \sqrt{s^2+t^2}$.

8. Find the equation of the tangent plane to the given surface, $z + 1 = xe^y \cos z$, at the point $(1,0,0)$.

9. Evaluate the integral $\int_0^1 \int_{3y}^3 e^{x^2}\,dxdy$.

10. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \dfrac{3^n(x+4)^n}{\sqrt{n}}$.

第二部份 計算證明題（40 %），（演算過程必須寫清楚，直接寫答案不計分，共四題，每小題十分）：

1. (a) State the Mean Value Theorem.

   (b) Show that the equation $1 + 2x + x^3 + 4x^5 = 0$ has exactly one real root.

2. Find the maximum and minimum of the function $f(x,y) = x^2y$ subject to the constraint $x^2 + 2y^2 = 6$.

3. Find the third-degree Taylor polynomial of function $f(x) = \sqrt[3]{x}$ at point $x = 8$.

4. Evaluate the integral $\iint_R \cos(\dfrac{y-x}{y+x})\,dA$, where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,2)$, and $(0,1)$.