淡江大學96學年度轉學生招生考試試題

系別：理工組二年級　　科目：微　積　分

可否使用計算機			
可	✓	否	

本試題共 1 頁

第一部份、(60 %) 簡答題 (不需寫出演算過程, 答案依照題號寫在答案卷第一頁, 題號要清楚標明, 共十題, 每小題六分):

(1) $\lim_{x\to 0} \frac{5-\sqrt{25-x}}{x} = ?$　　(2) $\lim_{x\to 0} \frac{(\sin 3x)^3}{x^2 \tan 5x} = ?$

(3) Let $f(x) = \frac{1}{1-x^3}$, $f'''(0) = ?$　　(4) Let $y = (1+x)^x$, $\frac{dy}{dx} = ?$

(5) Find an equation of the tangent line (切線) to the curve $x^2 + xy + 3y^2 = 15$ at the point $(1, 2)$.

(6) Find an equation of the tangent plane (切平面) to the surface $z = \sqrt{18 - 2x^2 - y^2}$ at the point $(2, 3, 1)$.

(7) $\int_0^1 xe^x dx = ?$

(8) Find the directional derivative of the function $f(x, y) = \ln(x^2 + y^2)$ at the point$(2, 1)$ in the direction of the vector $\vec{v} = -\vec{i} + 2\vec{j}$

(9) Find the area of the region (區域面積) bounded by $y = x$ and $y = \sqrt{x}$

(10) $\int_0^1 \int_{2y}^2 e^{x^2} dx dy = ?$

第二部份、(40 %) 計算題 (演算過程必須寫清楚, 直接寫答案不計分, 共四題, 每題十分):

(11) Sketch the graph (約略繪圖) of a function $f(x)$ that satifies the following conditions (滿足下列條件):

$f'(-2) = 0 = f'(2)$, $f'(x) < 0$ if $|x| < 2$ and $f'(x) > 0$ if $|x| > 2$

$f(-2) = 4$, $f(2) = 0$, $f''(x) < 0$ if $x < 0$ and $f''(x) > 0$ if $x > 0$.

(12) Find the maximum and minimum values (極大值與極小值) of the function $f(x, y) = 3x^2 + y^2$ on the region (在區域內) $x^2 + y^2 \le 1$.

(13) Find the radius of convergence (收斂半徑) and interval of convergence (收斂區間) of the series $\sum_{n=0}^{\infty} \frac{(-5)^n x^n}{\sqrt{n+1}}$.

(14) A right circular cylinder (圓柱體) is inscribed in (嵌入) a sphere (球體) of radius (半徑) 5 cm. Find the largest volume (最大體積) of such a cylinder.