淡江大學103學年度碩士班招生考試試題

系別：資訊工程學系
資訊工程學系資訊網路與通訊碩士班

科目：線性代數

考試日期：3月2日(星期日) 第3節

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本試題雙面印刷

1. (40%) Write **T** or **F** for each of the following statements to indicate whether the statement is true or false
   - (a) ____The set $\{(1,1,1)^T,(1,1,0)^T,(1,0,0)^T\}$ is a spanning set for R^3.
   - (b) ____The set $\{(1,1,1)^T,(1,1,0)^T,(1,0,0)^T\}$ is an orthogonal set in R^3.
   - (c) ____The vectors of $\{(1,2,1)^T,(2,9,0)^T,(3,3,4)^T\}$ are liner independent and span R^3.
   - (d) ____For any 3×3 square matrices A and B, $A\times B\neq B\times A$
   - (e) ____The distance from the point $(2,0,0)$ to the plane $x+2y+2z=0$ is $\frac{2}{3}$
   - (f) ____Let A be $n\times m$ matrix and λ be a scalar. "λ is an eigenvalues of A" and "$\det(A-\lambda I)=1$" are equivalent.
   - (g) ____Let A be $n\times m$ matrix, if A is non-singular, then A doesn't have a multiplicative inverse.
   - (h) ____Let A be a symmetric matrix, $A^T A=(A^T A)^T$.
   - (i) ____Let A be $n\times n$ symmetric matrix, if A has rank n, then the reduced row echelon form of A^T is identity matrix $I_{n\times n}$.
   - (j) ____Let $A=\begin{bmatrix}2&0&0\\0&4&0\\1&0&2\end{bmatrix}$ and $B=\begin{bmatrix}2&0&0\\-1&4&0\\-3&6&2\end{bmatrix}$. A and B both have the same eigenvalues.

2. (18%) Consider the matrices:

$$A=\begin{bmatrix}2&0\\-4&6\end{bmatrix}\quad B=\begin{bmatrix}1&-7&2\\5&3&0\end{bmatrix}\quad C=\begin{bmatrix}4&9\\-3&0\\2&1\end{bmatrix}\quad D=\begin{bmatrix}-2&1&8\\3&0&2\\4&-6&3\end{bmatrix}$$

$$E=\begin{bmatrix}0&3&0\\-5&1&1\\1&1&1\end{bmatrix}\quad F=\begin{bmatrix}1&0&0\\0&0&1\\0&1&0\end{bmatrix}\quad G=\begin{bmatrix}1&0&0\\0&1&0\\0&5&1\end{bmatrix}$$

   (a) (9%) Find the matrices X, Y, and Y^{-1} from matrices A~G such that

$$XY = \begin{bmatrix} -2 & 8 & 1 \\ 3 & 2 & 0 \\ 4 & 3 & -6 \end{bmatrix}.$$ (X, Y, and Y^{-1} should be selected from A~G)

(b) (3%) Find the matrices X from A~G such that $X = X^T$. (X should be selected from A~G)

(c) (6%) Find the matrices X and Y from A~G such that $CXY = \begin{bmatrix} 242 & 358 & -56 \\ -6 & 42 & -12 \\ 30 & 18 & 0 \end{bmatrix}$ (X and Y should be selected from A~G)

Show enough works to get full credits for the problem 3~5. (Answer alone get at most half credit.)

3. (12%) Determine the value of "a" for which the system of linear equations is (a) consistent (6%) and (b) inconsistent (6%)

$$\begin{aligned} x + 2y - 3z &= 2 \\ 2x - 2y + 3Z &= 1 \\ x + 2y - az &= a \end{aligned}$$

4. (15%) Find the least squares solution of the linear system $Ax = b$ where

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 3 \\ 2 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

5. (15%)

   (a) (10%) Find a point-normal form of the equation of the plane passing through point $p(1,1,1)$ and having vector $n = (2,0,0)$ as a normal.

   (b) (5%) Determine whether the given planes $3x - 2y + z = 6$ and $2x - y + 4z = 0$ are parallel