淡江大學 102 學年度碩士班招生考試試題

45-1

系別：資訊工程學系、
資訊工程學系資訊網路與通訊碩士班

科目：線性代數

考試日期：3 月 10 日(星期日)第 3 節　　本試題共 5 大題，1 頁

1. Write **T** or **F** for each of the following statements to indicate whether the statement is true or false, respectively. **(30%)**

(a) Every eigenvalue of an orthogonal matrix has absolute value 1.

(b) The points (2, 4, 6), (-2, 0, -1), (4, 1, -1), and (2, 0, -1) lie in the same plane.

(c) Let T: V → W be a one-to-one linear transformation, then *ker*(T) = V.

(d) Let $f = \cos^2 x$ and $g = \sin^2 x$, then the vectors $3x^2$ and 1 both lie in the space spanned by f and g.

(e) The number of subsets of S = {1, 2, 3, …,10} that contain the number 5 is 2^9.

(f) The coefficient of x^8 in the power series of the function $(1+x)(1+x^2)(1+x^3)(1+x^4)(1+x^5)$ is 3.

(g) Let A and B be two square matrices of the same size, then $det(A+B) = det(A) + det(B)$.

(h) There are 11! arrangements of letters of HOOPONOPONO.

(i) $3^{170} \bmod 29 \equiv 9$.

(j) $a_n = 3^n + (-6)^n$ is a solution to $a_n = -3a_{n-1} + 18a_{n-2}$.

For each of the following problem, show enough work to get full credits.

2. Prove by induction that for $n \geq 8$, $2^n \geq 3n^2 + 5$. **(19%)**

3. How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where for each i

(a) $x_i \geq 0$;

(b) $x_i \geq 1$;

(c) $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$;

(d) $x_i > i$. **(16%)**

4. Show that if A is a square matrix and $\|A\mathbf{u}\| = 1$ for all unit vectors $\mathbf{u}$, then A is orthogonal. **(17%)**

5. In each part, determine whether multiplication by A is a one-to-one linear transformation. **(18%)**

(a). $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 6 & -4 \end{bmatrix}$

(b). $A = \begin{bmatrix} 2 & 5 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$