系別：**資訊工程學系**
資訊工程學系資訊網路與通訊碩士班

科目：**數　學**(含離散數學、線性代數)

考試日期：2 月 28 日(星期一) 第 3 節　　　　本試題共 6 大題，　1 頁

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1. Prove that if x is irrational (無理數) then $\sqrt{x}$ is irrational. (16%)

2. How many different strings can be made by reordering the letters of the word *SUCCESS*? (17%)

3. What is the value of k after the following pseudocode has been executed? (17%)

```
k := 0
for i₁ := 1 to n
    for i₂ := 1 to i₁
        ⋮
            for iₘ := 1 to i_{m-1}
                k := k +1
```

$$k := 0$$
$$\textbf{for } i_1 := 1 \text{ to } n$$
$$\quad \textbf{for } i_2 := 1 \text{ to } i_1$$
$$\qquad \vdots$$
$$\qquad\quad \textbf{for } i_m := 1 \text{ to } i_{m-1}$$
$$\qquad\qquad k := k+1$$

4. Given that $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and det(A) = 6, find (16%)

(a). $\begin{vmatrix} g & a & d \\ h & b & e \\ i & c & f \end{vmatrix}$ 　　 (b). $\begin{vmatrix} -3a & -3b & -3c \\ d & e & f \\ g-4d & h-4e & i-4f \end{vmatrix}$ 　　 (c). det(A^2) 　　 (d). det($A+A$)

5. Determine whether the transformation T: $R^3 \rightarrow R^2$ is one-to-one. (14%)

$$T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 + 3x_3 \\ -x_1 - 4x_3 \end{bmatrix}$$

6. Given $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, 　 $b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$. (20%)

(a). Find a basis for the row space of A.

(b). Find a basis for the column space of A.

(c). Find a basis for the nullspace of A.

(d). Determine whether b is in the column space of A, and if so, express b as a linear combination of the column vectors of A.