

系別：資訊工程學系
資訊工程學系資訊網路與通訊碩士班

科目：數學(含離散數學、線性代數)

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁，5 大題

1. Fill in the blank or answer true/false.(36 pts)

- ___(a). Gram-Schmidt process can be performed on any nonempty set of linearly independent vectors in R^n .
- ___(b). Let A be a 3×3 matrix with characteristic equation $(\lambda+1)(\lambda-2)^2 = 0$. Then dimensions for the eigenspaces of A corresponding to the eigenvalues $\lambda = -1$ and $\lambda = 2$ are 1 and 2, respectively.
- ___(c). If U and V are vectors in R^n , then $|U \cdot V| \leq \|U\| \|V\|$.
- ___(d). Let W_1 and W_2 be subspaces in R^3 with equations $x-y+2z = 0$ and $x-y-z = 0$, respectively, then W_1 and W_2 are orthogonal complements.
- ___(e). Let A be an $n \times n$ matrix. The eigenvalues of A are the nonzero solutions of $\det(A - \lambda I) = 0$.
- ___(f). If the truth value for " $p \rightarrow q$ " is true then the truth value of " $q \rightarrow p$ " is ____.(choose one: true/false/not sure)
- ___(g). The area of a triangle with two sides given by $a = \langle 1, 3, -1 \rangle$ and $b = \langle 2, -1, 2 \rangle$ is ____.
- ___(h). If A is a 3×3 matrix such that $\det A = 5$, then $\det(1/2 A) + \det(-A^T) =$ ____.
- ___(i). If $\begin{pmatrix} a & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ b & 0 & \frac{1}{\sqrt{3}} \\ c & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ is orthogonal, then $a \cdot b \cdot c =$ ____.

For problems 2- 5, provide enough details to support your answer.
Answer alone will have at most half credits.

2. Prove by induction that $n^2 < n!$ for integer $n \geq 4$. (15 pts)

3. Find the least squares solution of the linear system $Ax = b$ given by

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$$

and find the orthogonal projection of b on the column space of A . (20 pts)

4. How many permutations of all 26 letters of the alphabet are there that contain none of the words: SAVE, PLAY, SNOW? (leave your answer in factorial form 答案保留 $n!$ 形式) (15 pts)

5. Find a MST (minimum spanning tree) by Kruskal's Algorithm. Label edges as ①, ②, ③, ..., according to the order that edges are chosen, and indicate the total weight of the corresponding MST. (14 pts)

