89-1

系別：資訊工程學系　　　　科目：數　學(含離散數學、線性代數)

准帶項目請打「V」	
X	簡單型計算機

本試題共 ／ 頁

1. Let W be the plane in R^3 with equation $5x - 3y + z = 0$.　　　**(25%)**
   (a). Find a basis for W.
   (b). Find the standard matrix for the orthogonal projection on W.
   (c). Use your answer to part (b) to find the orthogonal projection of the point $P_0(1, -2, 4)$ on W.
   (d). Find the distance between the point $P_0(1, -2, 4)$ and W.

2. Given $A = \begin{bmatrix} 4 & -5 \\ -3 & 2 \end{bmatrix}$　　　　　　**(25%)**
   (a). Find the characteristic equation of the matrix A.
   (b). Find the eigenvalues of the matrix A^{10}.
   (c). Find a basis for each eigenspace of the matrix A^{10}.
   (d). Use diagonalization to compute A^{10}.

3. Use Mathematical induction to prove that the following statement is true.　　　**(20%)**

   For all $n \in Z^+$, $n > 4 \Rightarrow n^2 < 2^n$.

4. How many ways are there to seat n different couples in a row so that at least one couple is seated next to each other? In a circle?　　**(15%)**

5. Let $S = \{3, 7, 11, 15, 19, ..., 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110?　　　**(15%)**