系別：資訊工程學系　　　　科目：數　學（含離散數學、線性代數）

1. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, please find (a) A^{-1}, (b) the eigenvalues and the corresponding eigenvectors for A, (c) A^{30}.

2. Let B be an idempotent matrix $B = \begin{bmatrix} 2/3 & b_1 & b_2 \\ b_1 & 2/3 & 1/3 \\ b_2 & 1/3 & b_3 \end{bmatrix}$,

   Please find (a) the rank of B, (b) $b_1^2 + b_2^2 + b_3^2$.

3. Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1) R (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$. Please
   (a) verify that R is an equivalence relation on A,
   (b) determine the equivalence classes $[(1, 3)], [(2, 4)], [(1, 1)]$,
   (c) determine the partition of A induced by R.

4. Let the Bernoulli numbers $b_0, b_1, b_2, \cdots$ be defined by the recurrence relation $b_n = \sum_{i=0}^{n} \binom{n}{i} b_{n-i}$, $n \geq 1$, with the boundary condition $b_0 = 1$. Please
   (a) determine the values of b_3, b_4 and b_5, (b) show that the exponential generating function of the sequence $b_0, b_1, b_2, \cdots$ is $\dfrac{x}{e^x - 1}$.

5. Given data :

y	1	4	8	10	3	8	9
x_1	−3	−2	−1	0	1	2	3
x_2	5	0	−3	−4	−3	0	5
x_3	1	−1	−1	0	1	1	−1

   Please find the regression curve $\hat{y} = a_0 + \dfrac{x_1}{a_1} + \dfrac{x_2}{a_2} + \dfrac{x_3}{a_3}$.

6. A computer dating service wants to match each of four girls with one of the six boys. According to the information they provided the following conclusions are drawn: (1) girl 1 would not be compatible with boy 1, 3, or 6.
   (2) girl 2 would not be compatible with boy 2, or 4.
   (3) girl 3 would not be compatible with boy 3, or 6.
   (4) girl 4 would not be compatible with boy 4, or 5.
   In how many ways can the service successfully match each of the four girls with a compatible boy?