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淡江大學九十二學年度碩士班招生考試試題

系別：資訊工程學系

科目：數學(含離散數學、線性代數)

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本試題共 / 頁

Show enough work to justify your answers!!

1. Prove by mathematical induction that

$$2^{n-1}(3^n + 4^n) > 7^n \text{ for all integers } n \geq 2. \quad (25\%)$$

2. How many integers between 1 and 100 are relatively prime to 100?
(The integer a is relatively prime to the integer b if and only if 1 is the only common positive divisor.) (25%)

3. Let $f(x) = \sin x$, $g(x) = \cos x$. Use the inner product

$$\langle f, g \rangle = \int_0^\pi f(x)g(x)dx, \text{ determine whether the set } \{f, g\} \text{ is orthonormal, orthogonal only, normal only, or neither.} \quad (25\%)$$

4. Find the eigenvalues and bases for the eigenspaces of A^{25} for

$$A = \begin{bmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix} \quad (25\%)$$