淡江大學九十學年度碩士班招生考試試題

系別：資訊工程學系　　科目：數　學（含離散數學、線性代數）

准帶項目請打「○」否則打「×」	
計算機	字典
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本試題共 1 頁

Show enough work to justify your answers!!

1. (18%) Build a generating function for a_r = the number of ways to distribute r similar balls into 5 numbered boxes with

(a). at most 3 balls in each box.

(b). at least 1 ball in each of the first 3 boxes and at least 3 balls in each of the last 2 boxes.

(c). a multiple of 5 balls in box 1, a multiple of 10 balls in box 2, a multiple of 25 balls in box 3, a multiple of 50 balls in box 4, and a multiple of 100 balls in box 5.

2. (16%) Use the pigeon-hole principle to show that one of any n consecutive integers is divisible by n.

3. (16%) Prove that $O(\log_a n) = O(\log_b n)$ where a and b are integers greater than 1.

4. (18%) Find the least squares solution of the given linear system:

$$\begin{aligned} 2x_1 + x_2 &= 26 \\ x_1 &= -13 \\ 2x_1 + 3x_2 &= 0 \end{aligned}$$

5. (16%) Consider R^3 with the Euclidean inner product. Use the **Gram-Schmidt** process to transform the given basis into an orthonormal basis.

$u_1 = (1,1,1), \quad u_2 = (1,1,0), \quad u_3 = (1,0,0)$

6. (16%) Consider the set of all real triples of the form (a,b,c), where $c = a - 2b$. Prove whether the set is a subspace of R^3.