淡江大學八十八學年度碩士班招生考試試題

系別：資訊工程學系　　科目：數學　　本試題共 1 頁

1. (%20) The complete binary tree $T = (V, E)$ has $V = \{a, b, c, d, e, f, g, h, i, j, k\}$. The postorder listing of V yields $d, e, b, h, i, f, j, k, g, c, a$. From this information draw T if (a) the height of T is 3; (b) the height of the left subtree of T is 3. Answer (a) and (b) by drawing two trees.

2. (%10) Prove or disprove that the following matrix has a LU-decomposition:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

3. (%24) Let $\Sigma_1 = \{w, x, y\}$ and $\Sigma_2 = \{x, y, z\}$ be alphabets. If $A_1 = \{x^i y^j \mid i, j \in Z^+, j > i \geq 1\}$, $A_2 = \{w^i y^j \mid i, j \in Z^+, i > j \geq 1\}$, $A_3 = \{w^i x^j y^i z^j \mid i, j \in Z^+, j > i \geq 1\}$, and $A_4 = \{z^j (wz)^i w^j \mid i, j \in Z^+, i \geq 1, j \geq 2\}$, determine whether each of the following statements is true or false (i.e., answer True or False).

a) A_1 is a language over Σ_1
b) A_1 is a language over Σ_2
c) A_2 is a language over Σ_1
d) A_2 is a language over Σ_2
e) A_3 is a language over $\Sigma_1 \cup \Sigma_2$
f) A_1 is a language over $\Sigma_1 \cap \Sigma_2$
g) A_4 is a language over $\Sigma_1 \triangle \Sigma_2$
h) $A_1 \cup A_2$ is a language over Σ_1

4. (%12) Prove or disprove that the following are linear combinations of $u = (0, -2, 2)$ and $v = (1, 3, -1)$:

a) $(2, 2, 2)$　b) $(0, 4, 5)$

5. (%24) Consider the following open statememt:

$$p(x, y) : y - x = y + x^2$$

where the universe for each of the variables x, y comprises all integers. Determine the truth value (i.e., answer True or False) for each of the following statememts:

$a) p(1, 1)$　$b) p(0, 3)$
$c) \forall y\ p(0, y)$　$d) \exists y\ p(1, y)$
$e) \forall x, y\ p(x, y)$　$f) \forall x \exists y\ p(x, y)$
$g) \exists y \forall x\ p(x, y)$　$h) \forall y \exists x\ p(x, y)$

6. (%10) a) Determine $P(G, \lambda)$ for $G = K_{1,3}$

b) For $n \in Z^+$, what is the chromatic polynomial for $K_{1,n}$? What is the chromatic number of $K_{1,n}$?