

淡江大學八十七學年度碩士班入學考試試題

系別：資訊工程學系

科目：數學(含離散數學、線性代數)

本試題共 / 頁

Show enough work to justify your answers!!

1. How many arrangements are there of the letters of the word MATHEMATICS with both T's before both A's, or both A's before both M's, or both M's before the E? (25%)

2. Solve the following recurrence relation using generating functions

$$a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0 \text{ for } n \leq 3 \text{ and } a_0 = 0, a_1 = 1, a_2 = -1. \quad (25\%)$$

3. Let P_2 be the set of all polynomial functions of degree ≤ 2 . Consider P_2 with the inner product $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$. Use the Gram-Schmidt process to transform the given basis $\{u_1, u_2, u_3\}$ into an orthogonal basis, where

$$u_1 = 1 + x^2, u_2 = 1 - x^2, u_3 = x. \quad (22\%)$$

4.

$$\text{Let } A = \begin{bmatrix} 5 & 3 & -7 \\ -1 & 1 & 1 \\ 3 & 3 & -5 \end{bmatrix}$$

(a). Find the eigenvalues of A .

(b). Find the basis for each eigenspace of A .

(c). Find a matrix P that diagonalizes A and determine $P^{-1}AP$.

(d). Use diagonalization to compute A^{10} . (28%)