淡江大學九十一學年度日間部轉學生招生考試試題

系別：理工組二年級　　　　　科目：微　積　分

准帶項目請打「○」否則打「×」
計　算　機
X

注意：第一題為填充題，請在答案卷第一頁依序寫上題號，再寫答案，不必寫出演算過程。第二、三、四、五題為計算、証明題，務必要有演算過程。

一、填充題（共10小題，每小題6分）

1. For $K \geq 0$, find $\lim\limits_{n \to \infty} \dfrac{1^K + 2^K + \cdots + n^K}{n^{K+1}}$.

2. Evaluate $\lim\limits_{x \to 1} \left(\dfrac{1}{x-1} - \dfrac{x}{\ln x} \right)$.

3. Find $(f^{-1})'(0)$, where $f(x) = \int_0^x \sqrt{1 + \cos^2 t}\, dt$.

4. Find $F'(0)$, where $F(x) = \int_{\sin x}^{e^x} \sqrt{1 + t^4}\, dt$.

5. Evaluate $\int_1^4 \dfrac{1}{\sqrt{x}(\sqrt{x}+1)^3}\, dx$.

6. Evaluate $\int_{-\infty}^{\infty} \dfrac{1}{x^2 + 2x + 10}\, dx$.

7. Find $g_y(\sqrt{5}, -2)$, where $g(x, y) = \tan^{-1} \dfrac{y^2}{x}$.

8. Find the sum of the series $\sum\limits_{k=2}^{\infty} \ln\left(1 - \dfrac{1}{k^2}\right)$.

9. Evaluate $\iint_S \sin(x^2 + y^2)\, dA$, where $S = \{(x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\}$.

10. Find the local extreme value(s) of
$$f(x, y) = xy + \dfrac{2}{x} + \dfrac{4}{y}.$$

◀ 注意背面尚有試題 ▶

（10分）二. Prove that if there exists a constant $M > 0$ such that

$$|f(y) - f(x)| \le M(y-x)^2$$

for all x and y, then f is a constant function.

（10分）三. (a) Write the Maclaurin series for $f(x) = \cos(x^2)$.

(b) Use part (a) to approximate $\int_0^1 \cos(x^2)\,dx$, accurate to three decimal places.

（10分）四. Let

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & \text{if } (x,y) \ne (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

(a) Show that f has a directional derivative in any direction at $(0,0)$.

(b) Is f continuous at $(0,0)$? Why?

(c) Is f differentiable at $(0,0)$? Why?

（10分）五. Find the volume of the solid cut from the ball $x^2 + y^2 + z^2 \le a^2$ by the cylinder $x^2 + y^2 = ay$.