

淡江大學九十學年度日間部轉學生招生考試試題

系別：理工組二年級

准帶項目請打「○」否則打「×」	
計算機	字典
×	×

科目：微 積 分

本試題共 2 頁

本試題雙面印製

注意：第一題為填充題，請在答案卷第一頁依序寫上題號再寫答案，不必寫出演算過程。第二、三、四、五題為計算、證明題，務必要有演算過程。

一、填充題（共 10 小題，每小題 6 分）

1. Evaluate $\lim_{h \rightarrow 0} \frac{(1+h)^{100} - 1}{h}$.
2. Find the point on the parabola $y^2 = 2x$ that is closest to the point (1, 4).
3. Consider the function $f(x) = x^3 + x + 1$. Find $(f^{-1})'(1)$.
4. Evaluate $\int_{-1}^1 \frac{\tan x}{1+x^2} dx$.
5. Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.
6. Evaluate the iterated integral $\int_0^1 \int_y^1 e^{x^2} dx dy$.
7. A curve C is defined by the parametric equations $x = t^2$ and $y = t^3 - 3t$. Find $\frac{d^2y}{dx^2}$ at $t = 1$.
8. For what values of a and b does the function $f(x) = x^3 + ax^2 + bx + 2$ have a local maximum when $x = -3$ and a local minimum when $x = -1$?
9. Evaluate $\int \ln x dx$.
10. Find $\lim_{n \rightarrow \infty} \frac{1}{n} \left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}} \right)$.

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(10分) 二、 (a) Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, where $f(x)$ is a continuous function.

(b) Use part (a) to show that $\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx = \frac{\pi}{4}$.

(10分) 三、 Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 3 \cdot 5 \cdots (2n-1)} x^n$.

(10分) 四、 If $z = y + f(x^2 - y^2)$, where f is differentiable, show that $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x$.

(10分) 五、 Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.