

系別：航空太空工程學系

科目：自動控制

考試日期：2 月 28 日(星期一) 第 2 節

本試題共

3 大題，

2 頁

1. Consider the lever-damper system shown in Figure 1, which consists of a mass,  $m$ , connected to an excitation base by a spring of stiffness  $k$  and two dampers in parallel with the same viscous damping coefficient  $c/2$ . Let  $z(t)$  and  $x(t)$  be the displacement of excitation base and the mass respectively. Moreover, assuming that (1) the **oscillations are small**, so that the **linear theory** is applicable, (2) gravitational force can be ignored.

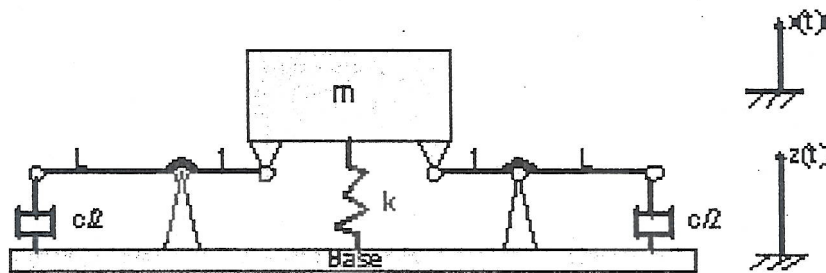


Figure 1. Lever-damper system

- (a) (10%) Derive the equation of motion of the system.
- (b) (5%) Determine the transfer function  $G(s) = X(s)/Z(s)$  of the system, where  $X(s)$  and  $Z(s)$  are the Laplace transform of  $x(t)$  and  $z(t)$  respectively.
- (c) (10%) Let  $m=2$  kg,  $k=1.5 \times 10^3$  N/m,  $c=36$  kg/s,  $L=2$  m, and assume that the base moves harmonically, i.e.,  $z(t) = 0.1 \sin(2t)$ ,  $t \geq 0$ . Find the **steady state** response of the mass  $m$ ,  $x_{ss}(t) = \lim_{t \rightarrow \infty} x(t)$ .
2. A **negative unity feedback** system has an open-loop transfer function  $G(s) = \frac{1.25}{s^2 + s + 1.25}$ .
- (a) (10%) The desired closed-loop performance specifications are: damping ratio  $\zeta = 0.707$ , natural frequency  $\omega_n = \sqrt{8}$ . Where should the closed-loop poles be located?
- (b) (10%) Design a **PD controller** that satisfies the above performance specifications. Also sketch the closed-loop system block diagram.
- (c) (5%) What will be the steady-state error of your compensated system to a **unit step** input?

CONTINUE

3. Consider the following closed-loop system,

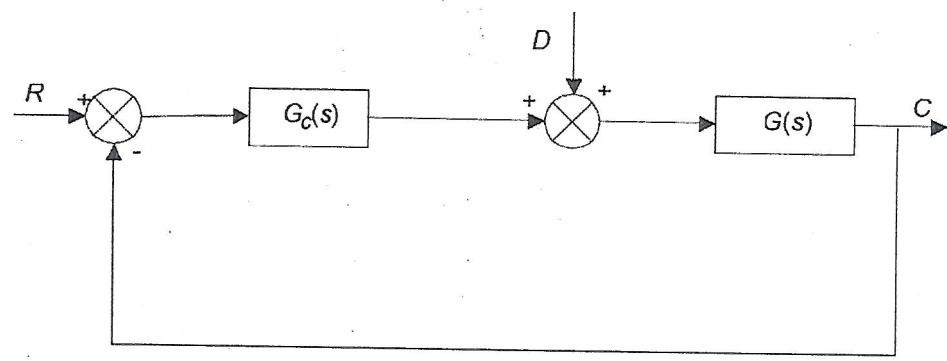


Figure 2 Close-loop system

where the controller  $G_c(s) = K$ , the system  $G(s)$  is represented by the pole-zero map shown Figure 3, and the steady state of the impulse response of the system is 1, that is  $\lim_{t \rightarrow \infty} g(t) = 1$ ,  $g(t) = \mathcal{L}^{-1}[G(s)]$ .

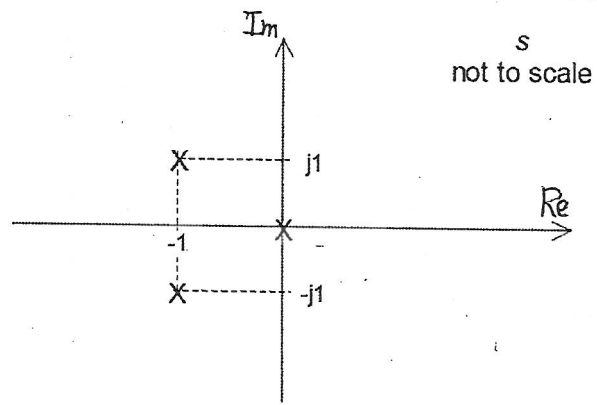


Figure 3 pole-zero map of  $G(s)$

- (a) (10%) Determine  $G(s)$  then draw the root-locus of the system for  $K > 0$ .
- (b) (10%) Calculate  $K_c$ , the value of  $K$  for which the closed-loop system becomes critically stable.
- (c) (10%) Sketch the complete Nyquist plot of  $G(s)$ . Clearly show  $0^+$ ,  $+j\infty$ ,  $-j\infty$ ,  $0^-$  and the direction of increasing frequency. Then indicate the range of  $K$  for a stable closed-loop system.
- (d) (10%) Consider a unity disturbance step input, i.e.  $R(s) = 0, D(s) = \frac{1}{s}$ . The design specifications require the steady-state step disturbance response  $c_{ss} = \lim_{t \rightarrow \infty} c(t) < 0.1$ . Will the proportional controller work? If so, give the controller design; if not, give an explanation.
- (e) (10%) For the controller obtained in part (c), use **straight line approximation** to draw the Bode diagram of  $G_c(s)G(s)$ . Then estimate the **gain margin** and **phase margin** of the closed-loop system.

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